

1.1 Power Diode

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* Power Diode Reverse Recovery

Key Formulas:

$$t_{rr} = t_a + t_b, \quad t_{rr} = \sqrt{\frac{2Q_{rr}}{dI/dt}}$$

$$SF = \frac{t_b}{t_a}$$

$$I_{rr} = \frac{dI}{dt} \times t_a$$

$$Q_{rr} = \frac{1}{2} I_{rr} t_a + \frac{1}{2} I_{rr} t_b = \frac{1}{2} I_{rr} (t_a + t_b) \\ = \frac{1}{2} I_{rr} t_{rr}$$

2082 Baisakh

1. For power diodes, the reverse recovery time is, $3.9 \mu s$ and the rate of diode current decay is $50 A/\mu s$. For a softness factor of 0.3 , calculate the peak inverse current I_{rr} and storage charge (Q_{rr}).

Given,

$$t_{rr} = 3.9 \mu s$$

$$\frac{dI}{dt} = 50 A/\mu s$$

$$SF = 0.3$$

We have,

$$t_{rr} = t_a + t_b$$

$$\text{or, } 3.9 = t_a + t_b \quad \text{--- (i)}$$

also,

$$SF = \frac{t_b}{t_a}$$

$$\text{or, } t_a = \frac{t_b}{0.3} \quad \text{--- (ii)}$$

From (i) and (ii), We get,

$$3.9 = \frac{t_b}{0.3} + t_b$$

$$\text{or, } 1.17 = t_b + 0.3 t_b$$

$$\text{or, } 1.17 = 1.3 t_b$$

$$t_b = 0.945$$

From (ii),

$$t_a = \frac{0.9}{0.3} = 345$$

Now,

For peak reverse recovery current, I_{rr} ,

$$I_{rr} = \frac{dQ}{dt} \times t_{rr}$$

$$= 50 \text{ A} \times 3 \mu\text{s}$$

$$\therefore I_{rr} = 150 \text{ A}$$

Again,

$$\text{For storage charge, } Q_t = \frac{1}{2} I_{rr} t_{rr}$$

$$= \frac{1}{2} \times 150 \text{ A} \times 3.9 \times 10^{-6} \text{ s}$$

$$\therefore Q_t = 292.5 \mu\text{C}$$

2071 Shrawan

- Q. A power diode has a reverse recovery time of $2.4 \mu\text{s}$. If dI/dt is $30 \text{ A}/\mu\text{s}$, find, i) stored charge ii) peak inverse current

given,

$$t_{rr} = 2.4 \mu\text{s}$$

$$\frac{dI}{dt} = 30 \text{ A}/\mu\text{s}$$

Now Assuming a linear reverse-recovery characteristic, the reverse-recovery current

Forms a triangular waveform.

Now,

For a triangular reverse recovery current,

$$I_{rr} = \frac{di}{dt} \times t_{rr}$$

$$= 30 \frac{A}{\mu s} \times 2.4 \mu s$$

$$\therefore I_{rr} = 72 A$$

also,

Stored charge,

$$Q_{rr} = \frac{1}{2} I_{rr} t_{rr}$$

$$= \frac{1}{2} \times 72 A \times 2.4 \mu s$$

$$\therefore Q_{rr} = 86.4 \mu C$$

Q.3 2079 Baisakhi

The reverse recovery time of a diode is $3 \mu s$ and the rate of fall of the diode current is $30 A/\mu s$. Determine storage charge and peak reverse current.

given,

$$t_{rr} = 8 \mu s$$

$$\frac{dI}{dt} = 30 A/\mu s$$

then,

for a linear reverse-recovery characteristics,
the reverse recovery current is given by -

$$I_{rr} = \frac{dI}{dt} \times t_{rr}$$

$$= 30 \frac{A}{\mu s} \times 8 \mu s$$

$$= 90 A$$

also,

stored charges,

$$Q_{rr} = \frac{1}{2} I_{rr} t_{rr}$$

$$= \frac{1}{2} \times 90 \times 8$$

$$= 360 \mu C$$

DC Chopper

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2076 Chaitra

1. ~~Ex~~ A step down DC chopper has a 20Ω resistor load and input voltage $220V$. When the converter switch is ON, its voltage drop in chopper switch is $1V$ and the chopper frequency is $2kHz$. If the duty cycle is 0.8 ,

calculate,

- i) Average value of the output voltage,
- ii) Efficiency of the chopper circuit.

Here, given,

$$R = 20\Omega$$

$$\text{Input voltage, } V_s = 220V$$

$$\text{Voltage drop across chopper switch, } V_{ch} = 1V$$

$$\text{Chopper frequency, } f = 2kHz$$

$$\text{Duty cycle} = 0.8$$

Then,

$$V_o (\text{during ON}) = V_s - V_{ch} = 220 - 1 = 219V$$

Average o/p voltage,

$$V_{avg} = K V_o$$

$$= 0.8 \times 219$$

$$= 175.2 \text{ V}$$

then,

$$I_{avg} = \frac{V_{avg}}{R} = \frac{175.2}{20} = 8.76 \text{ A}$$

then,

$$\rightarrow \text{Input power, } P_{in} = V_s I_{avg}$$

$$= 220 \times 8.76$$

$$= 1927.2 \text{ W}$$

~~$$\rightarrow \text{Output power, } P_{out} = V_{avg} \times I_{avg}$$~~

~~$$= 175.2 \times 8.76$$~~

~~$$= 1532.752 \text{ W}$$~~

$$\rightarrow \text{Output power, } P_{out} = \frac{V_{in}^2}{R}$$

$$= \frac{V_{rms}^2}{R}$$

$$= \frac{[\sqrt{2} (V_s - V_{ch})]^2}{R}$$

$$= \frac{(\sqrt{2} \times 219)^2}{20} = 1918.44 \text{ W}$$

Then,

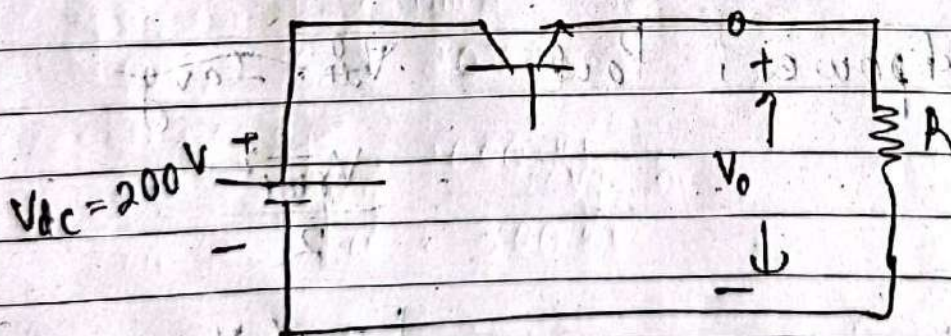
$$\text{Efficiency of chopper, } \eta = \frac{P_{out} \times 100\%}{P_{in}}$$

$$= \frac{1918.44}{1927.2} \times 100\%$$

$$= 99.55\%$$

2023 Chaitra

A step down DC chopper ($V_{dc} = 200V$) is operated in variable frequency mode with on time constant at 3ms. Calculate the OFF time for a duty cycle of 30% and average value of o/p voltage.



Given,

$$V_{dc} = 200V$$

$$t_{on} = 3msec$$

$$k = 30\%$$

We have,

$$k = \frac{t_{on}}{t_{on} + t_{off}}$$

$$\text{or, } 0.3 = \frac{3}{3 + t_{off}}$$

$$\text{or, } 3 + t_{off} = 10$$

$$\therefore t_{off} = 10 - 3 = 7msec$$

also,

$$V_{avg} = k V_{dc}$$

$$= 0.3 \times 200$$

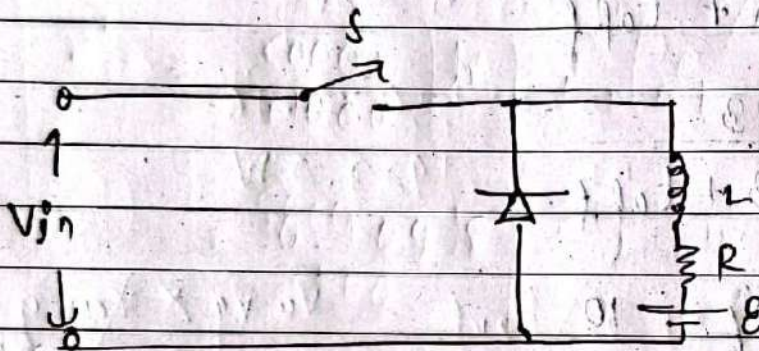
$$= 60V$$

2079 Bairath

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Q. A step down chopper has an input of $200V$, it is feeding on RLE load having $R = 2\Omega$, $L = 10mH$, $E = 20V$. The chopping cycle has a time period of $1000\mu s$ and transistor is on for $300\mu s$ in each cycle. (i) Find whether the load current is continuous or not (ii) Find the average load current.



given,

$$V = 200V$$

$$R = 2\Omega$$

$$L = 10mH$$

$$E = 20V$$

$$T = 1000\mu s$$

$$t_{on} = 300\mu s$$

Key formulas

$$\tau = \frac{L}{R} \text{ (time constant)}$$

$$m = \frac{E}{V} \text{ (EMF ratio)}$$

$$D' = \frac{\tau}{T} \ln \left[1 + m \left(e^{T/\tau} - 1 \right) \right] \text{ (critical duty cycle)}$$

$$K = \frac{T_{on}}{T}$$

Condition: $D > D'$ (continuous)

$D < D'$ (discontinuous)

then,

$$I_{avg} = \frac{KV - E}{R} = \frac{V_{avg} - E}{R}$$

Solution

$$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}}{2} = 5000 \mu s$$

$$m = \frac{E}{V} = \frac{20}{200} = 0.1$$

$$D = K = \frac{T_{on}}{T} = \frac{300}{1000} = 0.3$$

$$\frac{T}{\tau} = \frac{1000}{5000} (1 + 0.2)$$

Now,

Critical duty cycle

$$D' = \frac{\tau}{T} \ln \left[1 + m \left(e^{T/\tau} - 1 \right) \right]$$

$$= \frac{5000}{1000} \ln \left[1 + 0.1 \left(e^{0.2} - 1 \right) \right]$$

$$= 0.1095$$

Here,

$$D = 0.3 > D' (= 0.1095)$$

Hence, the load current is continuous.

Now,

$$I_{avg} = \frac{V_{avg} \cdot E}{R} = \frac{K V_{in}^0 - E}{R}$$

$$= \frac{0.3 \times 200 - 20}{2}$$

$$I_{avg} = 20 \text{ A}$$

2081 Bhadra

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4. For A type - A chopper circuit, source voltage $V_s = 220V$, chopping period $T = 2000\mu s$, on-period = $600\mu s$, load circuit parameters: $R = 1\Omega$, $L = 5mH$ and $E = 24V$

- i) Find whether load current is continuous or not
- ii) Calculate the value of avg. o/p. current
- iii) Compute the maximum and minimum values of steady state o/p. current
- iv) sketch the time variations of load voltage V_o , load current I_o , thyristor current I_T and voltage across thyristor V_T .

given,

$$V_s = 220V$$

$$T = 2000\mu s$$

$$T_{on} = 600\mu s$$

$$R = 1\Omega$$

$$L = 5mH$$

$$E = 24V$$

Solution,

$$\tau = \frac{L}{R} = \frac{5mH}{1\Omega} = 5mSec = 5000\mu s$$

$$m = \frac{E}{V} = \frac{24}{220} = 0.1091$$

$$K = \frac{I_{on}}{I} = \frac{600}{2000} = 0.3$$

$$K' = \frac{I_{off}}{I} = \frac{2000}{5000} = 0.4$$

Now,

Critical duty cycle,

$$K' = \frac{\tau}{T} \ln \left[1 + \frac{m}{1-m} (e^{\tau/T} - 1) \right]$$

$$= \frac{5000}{2000} \ln \left[1 + \frac{24}{220} (e^{0.4} - 1) \right]$$

$$= 0.13066$$

Here,

$K > K'$. So, the load current is continuous.

Now,

$$I_{avg} = \frac{V_{avg} - E}{R}$$

$$= \frac{0.3 \times 220 - 24}{1}$$

$$= 1 \text{ A}$$

$$= 42 \text{ A}$$

ii) The max^m value of load current I_L

$$I_{\max} = \frac{V \left(1 - e^{-\frac{T_{ON}}{\tau}} \right)}{R \left(1 - e^{-T/\tau} \right)} - \frac{E}{R}$$

$$= \frac{220 \left(1 - e^{-\frac{600}{5000}} \right)}{1 \left(1 - e^{-0.4} \right)} - \frac{24}{1}$$

$$= 51.46 \text{ A}$$

The min^m value of load current I_L ,

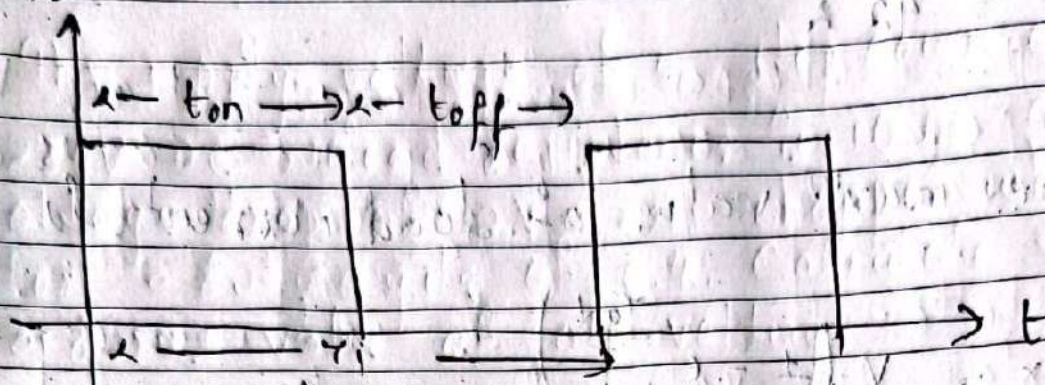
$$I_{\min} = \frac{V \left(e^{-T_{ON}/\tau} - 1 \right)}{R \left(e^{-T/\tau} - 1 \right)} - \frac{E}{R}$$

$$= 33.031 \text{ A}$$

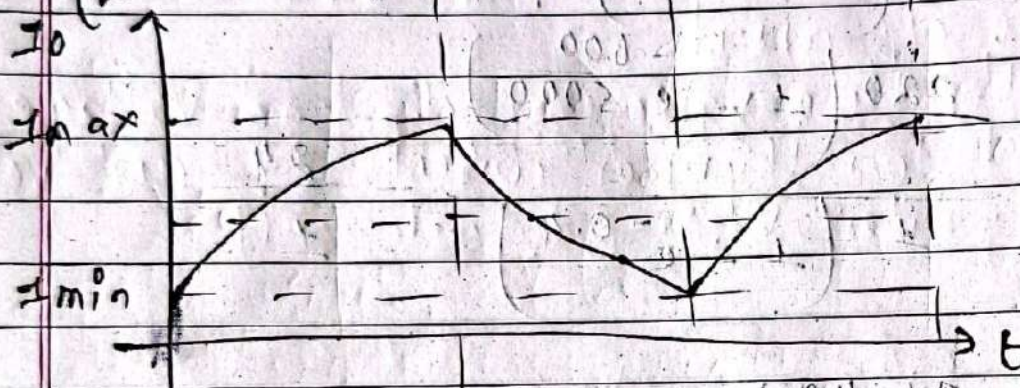
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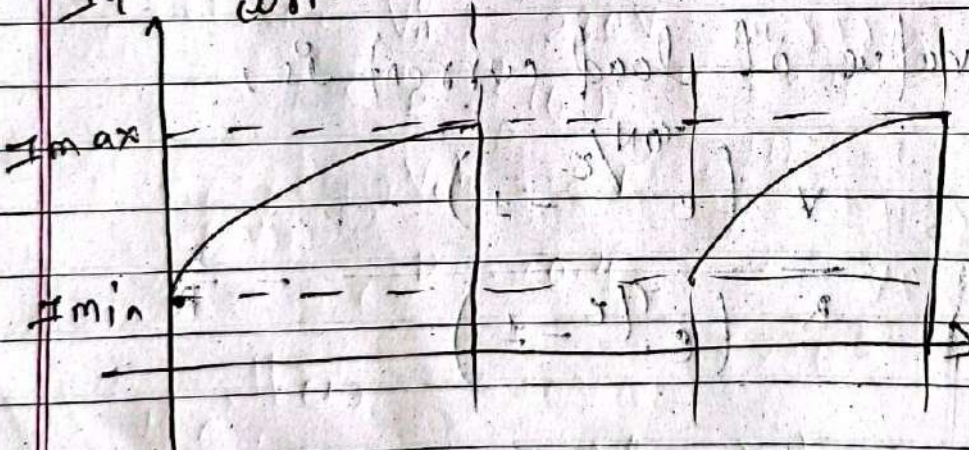
V_o (o/p Voltage)



I_o (Load current)



I_t (thyristor current)



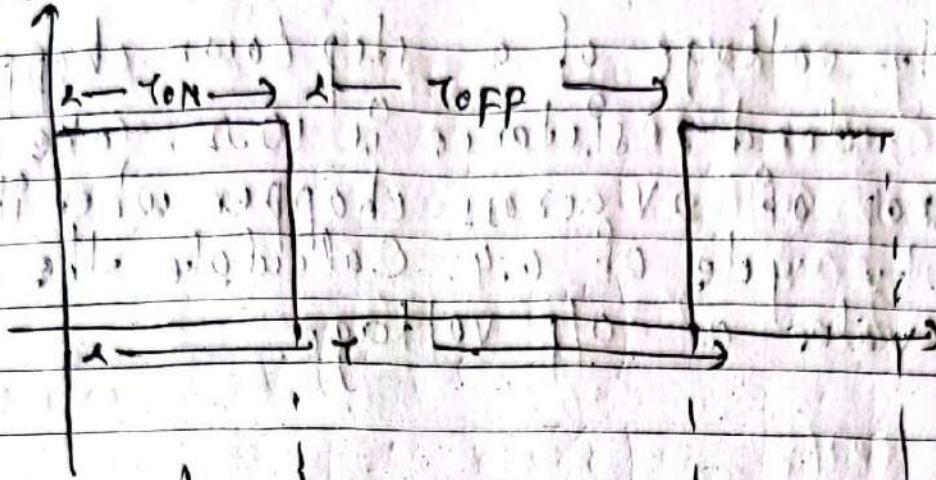
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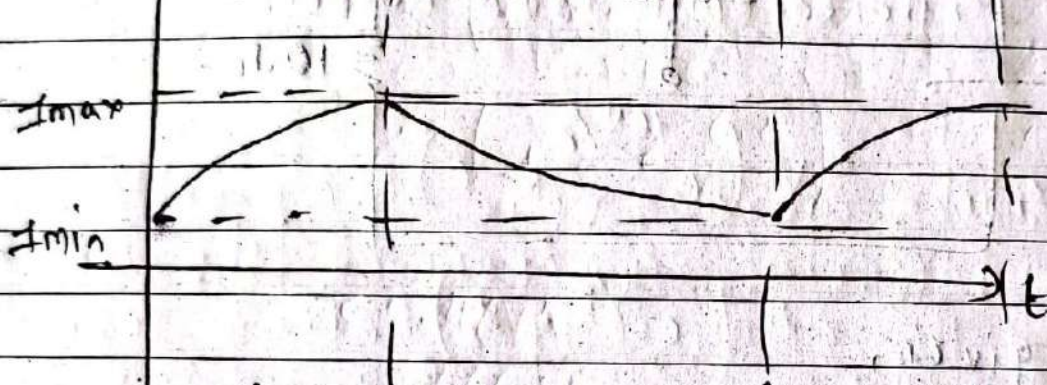
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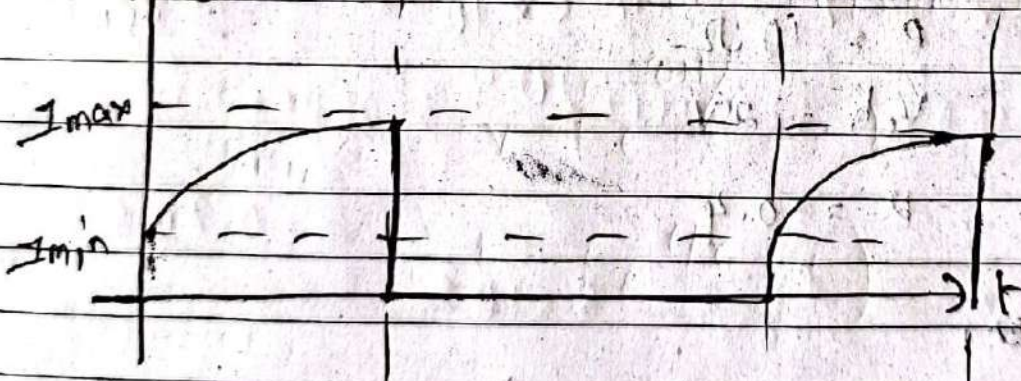
V_o (output voltage)



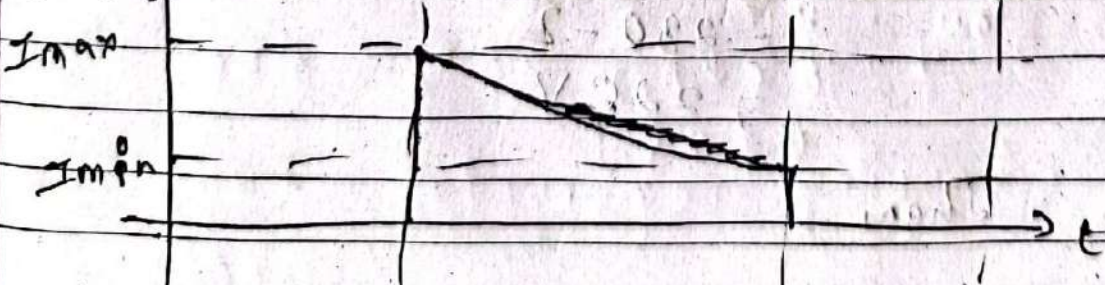
I_L (Load current)



I_T (thyristor current)

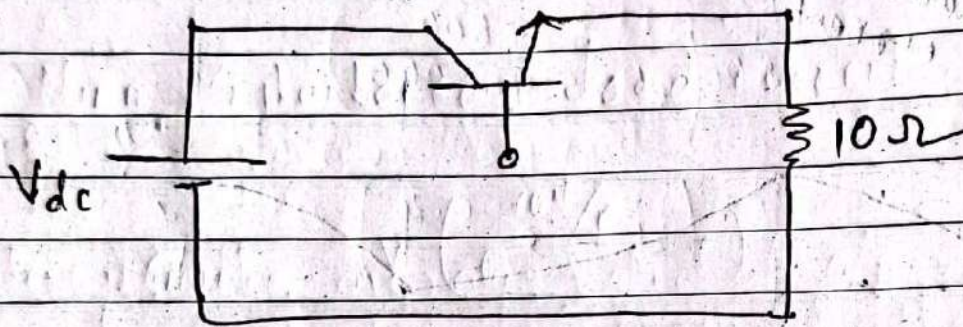


I_{fd}



2070 Chaitra

- 5) The supply voltage of a step down chopper is 230V DC, load resistance is 10Ω . Take the voltage drop of 2V across chopper when it is on. For a duty cycle of 0.4. Calculate the avg and rms value of o/p voltage.



Here, given,

$$V_{in} = 230V$$

$$R = 10\Omega$$

$$V_{ch} = 2V$$

$$k = 0.4$$

Then,

$$\begin{aligned} V_o &= V_{in} - V_{ch} \\ &= 230 - 2 \\ &= 228V \end{aligned}$$

Then,

$$\begin{aligned}
 V_{avg} &= K V_0 \\
 &= 0.4 \times 220 \\
 &= 88 \text{ V}
 \end{aligned}$$

Then,

$$\begin{aligned}
 V_{rms} &= \sqrt{K} V_0 \\
 &= \sqrt{0.4} \times 220
 \end{aligned}$$

$$= 144.199 \text{ V}$$

2079 Bhadra

- 6) A step down chopper is connected to RLE load with $R = 10 \Omega$, $L = 15.5 \text{ mH}$ and $E = 20 \text{ V}$. with an input voltage of 220 V , the duty cycle $K = 0.5$ and chopping frequency, $F = 5 \text{ kHz}$.

Determine,

- i) Minimum instantaneous load current
- ii) Peak instantaneous load current
- iii) Maximum peak to peak current in the load
- iv) Avg load current

given,

$$V_{in} = 220V$$

$$R = 10\Omega$$

$$L = 15.5mH$$

$$E = 20V$$

$$R = 0.5$$

$$f = 5kHz$$

i) Minm Instantaneous load current,

$$I_{min} = \frac{V \left(e^{\tau_{ON}/\tau} - 1 \right)}{R \left(e^{\tau/\tau} - 1 \right)}$$

here,

$$\tau = \frac{L}{R} = \frac{15.5 \times 10^{-3}}{10} = 1.55ms$$

then,

$$\tau = \frac{1}{f} = \frac{1}{5 \times 10^3} = 0.2ms$$

then,

$$\begin{aligned} \tau_{ON} &= K\tau \\ &= 0.5 \times 0.2 \\ &= 0.1ms \end{aligned}$$

Then,

$$I_{min} = \frac{220 \left(e^{0.1/1.55} - 1 \right)}{10 \left(e^{0.2/1.55} - 1 \right)} - \frac{20}{10}$$

$$= 10.645 - 2$$

$$= 8.645 \text{ A}$$

ii) Peak instantaneous load current

$$I_{max} = \frac{V \left(1 - e^{-\tau/\tau_c} \right)}{R \left(1 - e^{-T/\tau_c} \right)} + \frac{\mathcal{E}}{R}$$

$$= 9.355 \text{ A}$$

iii) Maximum peak to peak current in the load
(ΔI_{max})

i.e.

$$\Delta I_{max} = I_{max} - I_{min}$$

$$= 9.355 - 8.645$$

$$= 0.71 \text{ A}$$

ii) Avg. Load current (I_{avg})

$$I_{avg} = \frac{V_{avg} - E}{R}$$

$$= \frac{0.5 \times 220 - 20}{10}$$

$$= 9A$$

2075 Ashwin

Determine the average value of o/p voltage and the fundamental component of the load current of step down chopper having input voltage of 220V DC, load resistance 20Ω and duty cycle of 45%.

Given, chopping frequency of chopper is $1kHz$.

Given, $V_{in} = 220V$

$$V_{in} = 220V$$

Load resistance, $R = 20\Omega$

$$\text{Duty cycle } (\alpha) = 45\% = 0.45$$

$$\text{Chopping frequency } (f) = 1kHz$$

$$= 1000Hz$$

$$1) V_{avg} = k V_{in}$$

$$= 0.45 \times 220$$

$$= 99 \text{ V}$$

ii) Fundamental component of load current

The n th harmonic peak voltage amplitude of a step down chopper waveform is given by Fourier analysis:

$$V_{nm} = \frac{2V_{in}}{n\pi} \sin(n\pi k)$$

For the fundamental component ($n=1$):

$$V_{1m} = \frac{2 \times 220}{\pi} \sin(0.45\pi)$$

$$= 138.33 \text{ V (peak)}$$

⇒ RMS Fundamental Voltage,

$$V_{1,rms} = \frac{V_{1,m}}{\sqrt{2}} = \frac{138.33}{\sqrt{2}}$$

$$\therefore V_{1,rms} = 97.814 \text{ V}$$

Then,

→ Peak Fundamental Load Current (I_{1m}),

$$I_{1m} = \frac{V_{1m}}{R} = \frac{138.33}{20}$$

$$I_{1m} = 6.9165 \text{ A}$$

→ RMS Fundamental Load Current (I_{1rms}),

$$I_{1rms} = \frac{V_{1rms}}{R}$$

$$I_{1rms} = \frac{97.814}{20}$$

$$I_{1rms} = 4.8907 \text{ A}$$

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$$I_{1rms} = 4.8907 \text{ A}$$

$$I_{1rms} = 4.8907 \text{ A}$$

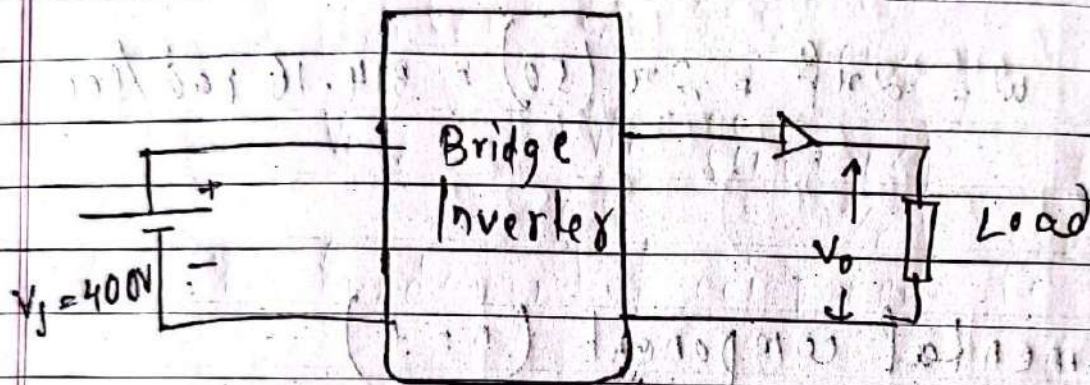
$$I_{1rms} = 4.8907 \text{ A}$$

$$I_{1rms} = 4.8907 \text{ A}$$

InverterDate _____
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2079 Shrawan

Q. A single phase full bridge inverter with DC input voltage of $V_s = 400V$ and generating o/p square wave of $50Hz$ is connected to inductive load having $R = 10\Omega$ and $L = 50mH$. Calculate the magnitude and phase of fundamental component and third harmonic component of o/p voltage and load current.



Given,

$$V_s = 400V$$

$$f = 50Hz$$

$$R = 10\Omega, L = 50mH$$

Here,

For a single-phase full bridge inverter producing a square wave o/p of amplitude V , the Fourier series contains only the odd harmonics:

$$v_o(t) = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin(n\omega t)$$

So, the peak value of n th harmonic voltage

$$V_{on} = \frac{4V_s}{n\pi}$$

Fundamental angular frequency:

$$\omega = 2\pi f = 2\pi(50) = 314.16 \text{ rad/sec}$$

Then,

⇒ Fundamental component ($n=1$)

→ Peak value:

$$V_{o1} = \frac{4V_s}{\pi} = \frac{4(400)}{\pi}$$

$$= 509.30 \text{ V}$$

→ RMS value:

$$V_{orms} = \frac{V_{o1}}{\sqrt{2}} = \frac{509.30}{\sqrt{2}}$$

$$= 360.18 \text{ V}$$

Thus the fundamental voltage is,

$$\Rightarrow v_{o1}(t) = 509.30 \sin(314.16t) \text{ V}$$

or, in phasor form,

$$V_{o1} = 360.13 \angle 0^\circ \text{ V (rms)}$$

→ Load impedance at fundamental frequency,

$$X_L = \omega L = 314.16 \times 50 \times 10^{-3} = 15.71 \Omega$$

$$Z_1 = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{10^2 + 15.71^2}$$

$$= 18.623 \Omega$$

phase ↙

Impedance angle :

$$\theta_1 = \tan^{-1} \left(\frac{X_L}{R} \right) = \tan^{-1} \left(\frac{15.71}{10} \right)$$

$$= 57.52^\circ$$

Hence, the fundamental current RMS magnitude is,

$$I_{01} = \frac{V_{01, \text{rms}}}{|Z_1|}$$

$$= \frac{960.13}{18.623}$$

$$I_{01} = 19.34 \text{ A rms}$$

Therefore,

$$I_{01} = 19.34 \angle -57.52^\circ \text{ A}$$

Peak current:

$$I_{01, \text{peak}} = \sqrt{2} \times 19.34$$

$$= 27.35 \text{ A}$$

⇒ So, the time domain fundamental current is,

$$i_{01}(t) = 27.35 \sin(1314.16t - 57.52^\circ) \text{ A}$$

2) Third harmonic component of o/p voltage

For $n=3$

$$V_{03} = \frac{4V}{3\pi}$$

$$V_{03} = \frac{4(400)}{3\pi}$$

$$= 169.77 \text{ V (peak)}$$

→ RMS,

$$V_{03, \text{rms}} = \frac{169.77}{\sqrt{2}} = 120.046 \text{ V}$$

Thus the fundamental voltage & third harmonic component of the voltage is,

$$v_{03}(t) = \frac{169.77}{169.77} \sin(3\omega t)$$

$$\therefore v_{03}(t) = 120.046 \sin(942.48t) \text{ V}$$

then,

$$V_{03} = 120.046 \angle 0^\circ \text{ V (rms)}$$

=) Third harmonic component of load current

At the third harmonic,

$$X_{L3} = 3\omega L$$

$$= 942.48 \times 50 \times 10^{-3}$$

$$= 47.124 \Omega$$

Then,

$$Z_3 = \sqrt{R^2 + X_{L3}^2}$$

$$= \sqrt{10^2 + 47.124^2}$$

$$= 48.173 \Omega$$

Phase angle

$$\phi_3 = \tan^{-1} \left(\frac{X_{L3}}{R} \right)$$

$$= \tan^{-1} \left(\frac{47.124}{10} \right)$$

$$= 78.019^\circ$$

then,

$$I_{03} = \frac{V_{03, \text{rms}}}{|Z_3|}$$

$$= \frac{120.046}{48.173}$$

$$= 2.492 \text{ A rms}$$

then,

$$I_{03} = 2.492 \angle -78.02^\circ \text{ A}$$

Peak current,

$$I_{03, \text{peak}} = 2.492 \times \sqrt{2}$$

$$= 3.524 \text{ A}$$

thus,

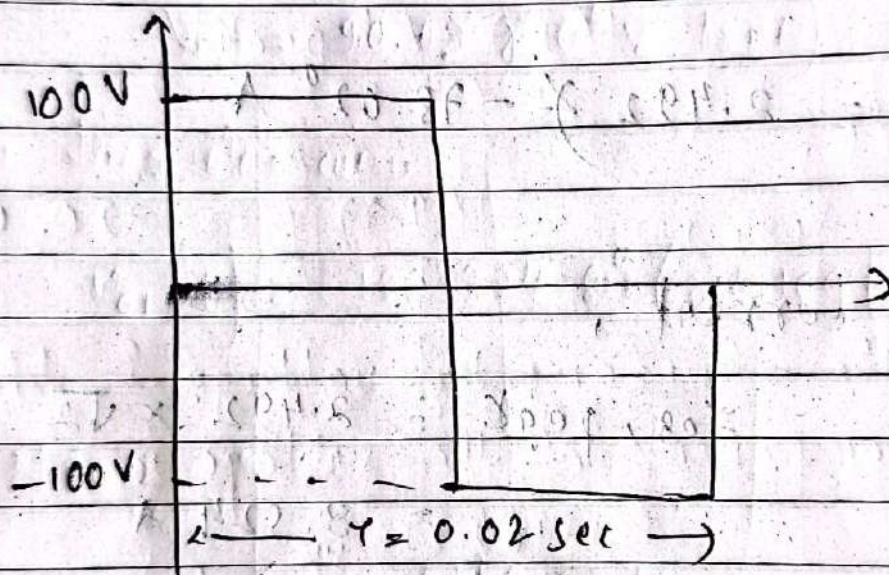
$$i_{03}(t) = 3.524 \sin(942.48t - 78.02^\circ) \text{ A}$$

2020 Asadh

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- Q. A single phase inverter output voltage waveform (square wave, $\pm 100V$, $T = 0.02 \text{ sec}$) with inductive load $R = 10\Omega$ and $L = 20 \text{ mH}$ connected in series. Calculate the magnitude and phase of fundamental component of the o/p voltage and write down the time domain equation of fundamental component of load current.



Given,

$$V = 100V$$

$$T = 0.02 \text{ sec}$$

$$R = 10\Omega$$

$$L = 20 \text{ mH}$$

Here,

$$f = \frac{1}{T} = \frac{1}{0.02} = 50 \text{ Hz}$$

then,

$$\begin{aligned}\omega &= 2\pi f \\ &= 2\pi \times 50 \\ &= 314.159 \text{ rad/sec}\end{aligned}$$

For a symmetrical square wave of amplitude V_s , the Fourier series contains only odd harmonics

$$v_o(t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_s}{n\pi} \sin(n\omega t)$$

For fundamental component ($n=1$)

$$v_{o1}(t) = \frac{4V_s}{\pi} \sin(\omega t)$$

For the peak value,

$$V_{o1} = \frac{4 \times 100}{\pi}$$

$$= 127.324 \text{ V}$$

Then,

$$V_{01, rms} = \frac{127.324}{\sqrt{2}}$$

$$= 90.09 \text{ V}$$

Then,

The fundamental voltage phasor is,

$$V_{01} = 90.09 \angle 0^\circ \text{ V (rms)}$$

In time domain,

$$v_{01}(t) = 127.324 \sin(314.159t) \text{ V}$$

Now,

$$X_L = \omega L$$

$$= 314.159 \times 20 \times 10^{-3}$$

$$= 6.283 \Omega$$

Then,

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{10^2 + 6.283^2}$$

$$= 11.81 \Omega$$

Then,

$$I_{01, \text{rms}} = \frac{V_{01, \text{rms}}}{Z}$$

$$Z = 12.1$$

$$= \frac{90.03}{11.81}$$

$$= 7.623 \text{ A}$$

Then,

$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$= \tan^{-1} \left(\frac{6.283}{10} \right)$$

$$= 32.14^\circ$$

Then,

$$I_{01} = 7.623 \angle -32.14^\circ \text{ A (rms)}$$

Then,

$$I_{01, \text{peak}} = 7.623 \times \sqrt{2}$$

$$= 10.781 \text{ A}$$

Then,

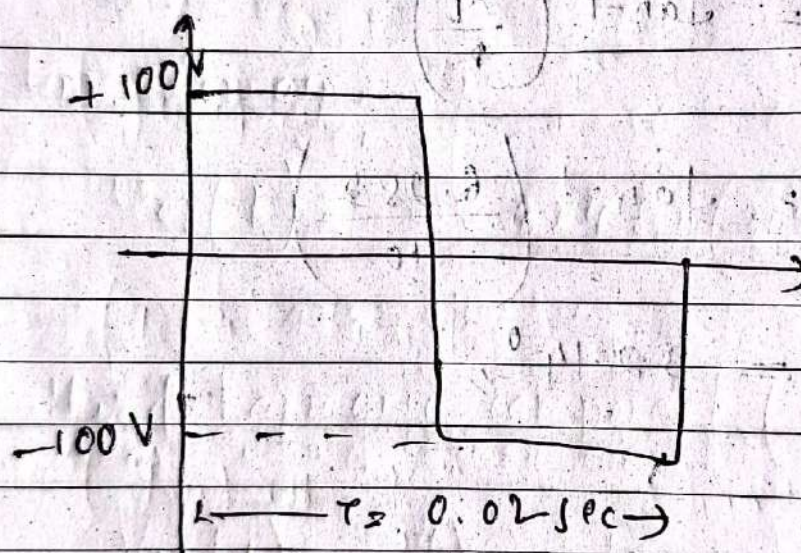
$$i_{01}(t) = 10.781 \sin(314.159t - 32.14^\circ)$$

A

2073 chaira

A single phase inverter output voltage waveform with inductive load $R = 5\Omega$ and $L = 10\text{mH}$ connected in series.

Calculate the magnitude and phase of fundamental component of the o/p voltage and write the time domain equation of the current considering Fourier series up to 5th order.



given,

$$V_s = 100\text{V}$$

$$T = 0.02\text{sec}$$

$$R = 5\Omega$$

$$L = 10\text{mH}$$

then,

$$f = \frac{1}{T} = \frac{1}{0.02} = 50 \text{ Hz}$$

then,

Fourier series for

For a symmetrical square wave of amplitude V_s , the Fourier series contains only the odd harmonics.

$$v_o(t) = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin(n\omega t)$$

For fundamental component ($n=1$)

$$v_{o1}(t) = \frac{4V_s}{\pi} \sin(\omega t)$$

peak value is given by,

$$V_{o1} = \frac{4V_s}{\pi}$$

$$= \frac{4 \times 100}{\pi}$$

$$= 127.324 \text{ V}$$

Then,

$$V_{01, rms} = \frac{V_{01}}{\sqrt{2}}$$

$$= \frac{127.324}{\sqrt{2}}$$

$$= 90.032 \text{ V}$$

Then,

$$V_{01} = 90.032 \text{ V (rms)}$$

In time domain,

$$V_{01}(t) = 127.324 \sin(314.159t)$$

2) For third harmonic component ($n=3$)

$$V_{03}(t) = \frac{4V_s}{3\pi} \sin(3\omega t)$$

Then,

$$\text{peak, } V_{03} = \frac{4 \times 100}{3\pi}$$

$$= 42.44 \text{ V}$$

also,

$$V_{03, \text{rms}} = \frac{42.44}{\sqrt{2}}$$

$$= 30.01 \text{ V}$$

⇒ For 5th harmonic component ($n=5$)

$$V_{05}(t) = \frac{4V_s}{5\pi} \sin(5\omega t)$$

peak,

$$V_{05} = \frac{4 \times 100}{5\pi} = 25.465 \text{ V}$$

also,

$$V_{05, \text{rms}} = \frac{25.465}{\sqrt{2}} = 18.0064 \text{ V}$$

⇒ Now for load current,

$$I_{01, \text{rms}} = \frac{V_{01, \text{rms}}}{Z_1}$$

then,

$$Z_1 = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{5^2 + (2\pi \times 50 \times 10 \times 10^{-3})^2}$$

$$= 5.91 \Omega$$

Then,

$$I_{01, \text{rms}} = \frac{90.032}{5.91}$$

$$I_{01, \text{rms}} =$$

$$= 15.24 \text{ A}$$

$$I_{01, \text{peak}}$$

$$I_{01, \text{peak}} = 5.91 \times \sqrt{2} \quad 15.24 \times \sqrt{2}$$

$$= 8.345 \text{ A} \quad = 21.55 \text{ A}$$

then,

$$\phi_1 = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$= \tan^{-1} \left(\frac{2\pi \times 50 \times 10 \times 10^{-3}}{5} \right)$$

$$= 32.142^\circ$$

=) Now for third harmonic component,

$$I_{03, \text{rms}} = \frac{V_{03, \text{rms}}}{Z_3}$$

$$Z_3$$

$$= 30.01 \Omega$$

$$\sqrt{5^2 + (6\pi \times 50 \times 10 \times 10^{-3})^2}$$

$$(0.01 \times 0) = 2.813 \text{ A}$$

then,

$$I_{00, \text{peak}} = 2.813 \times \sqrt{2}$$

$$= 3.9722 \text{ A}$$

then,

$$\theta_3 = \tan^{-1} \left(\frac{3X_L}{R} \right)$$

$$= \tan^{-1} \left(\frac{6\pi \times 50 \times 10 \times 10^{-3}}{5} \right)$$

$$= 62.053^\circ$$

⇒ Now for fifth harmonic component,

$$I_{05, \text{rms}} = \frac{V_{05, \text{rms}}}{Z_5}$$

$$= \frac{18.0064}{25}$$

$$I_{05, \text{peak}} = \frac{V_{05, \text{peak}}}{Z_5}$$

$$= \frac{25.465}{\sqrt{5^2 + (10\pi \times 50 \times 10^{-2})^2}}$$

$$= 1.545 \text{ A}$$

then,

$$\theta_s = \tan^{-1} \left(\frac{5X_L}{R} \right)$$

$$= \tan^{-1} \left(\frac{10\pi \times 50 \times 10^{-2}}{5} \right)$$

$$= 72.343^\circ$$

then,

$$i_o(t) = 21.55 \sin(314.159t - 32.142^\circ)$$

$$+ 3.9782 \sin(942.48t - 62.053^\circ)$$

$$+ 1.545 \sin(1570.796t - 72.343^\circ)$$

A single phase Inverter with resistive load $R_o = 5\Omega$ and $L_o = 20mH$ and the input voltage is 200V DC with a centre point of DC source grounded. The Inverter is operated to give an o/p AC voltage of 50Hz. Find the time domain equation of the load current using Fourier series up to 3rd order. Also find the harmonic factor and THD of o/p voltage.

given,

$$V_{dc} = 200V \quad (\text{with center tapped / grounded mid point})$$

$$R_o = 5\Omega$$

$$L_o = 20mH$$

$$f = 50Hz$$

Here,

$$v_o(t) = \begin{cases} +100V & 0 < t < T/2 \\ -100V & T/2 < t < T \end{cases}$$

thus, the voltage magnitude is,

$$V = \frac{V_{dc}}{2} = 100V$$

For a symmetrical square wave, only odd harmonics are present.

So,

$$v_o(t) = \frac{4V}{\pi} \left[\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \dots \right]$$

Here,

$$V = 100 \text{ V}$$

$$v_o(t) = \frac{4 \times 100}{\pi} \left[\sin \omega t + \frac{1}{3} \sin 3\omega t + \dots \right]$$

$$\therefore v_o(t) = 127.33 \sin(314.159t) + 42.44 \sin(942.48t)$$

(up to 3rd harmonic)

Then,

Now,

$$|Z_n| = \sqrt{R^2 + (\omega L)^2}$$

For $n = 1$,

$$Z_1 = \sqrt{5^2 + (2\pi \times 50 \times 20 \times 10^{-3})^2}$$

$$= 8.029 \Omega$$

then,

$$I_{01}, \text{ peak} = \frac{127.33}{8.029} \\ = 15.86 \text{ A}$$

$$\theta_1 = \tan^{-1} \left(\frac{2\pi \times 50 \times 20 \times 10^{-3}}{5} \right)$$

$$= 51.49^\circ$$

Similarly,

for,

I_{02} , peak

$$Z_2 = \sqrt{5^2 + (3 \times 2\pi \times 50 \times 20 \times 10^{-3})^2} \\ = 19.50 \Omega$$

then,

$$I_{02}, \text{ peak} = \frac{42.44}{19.50} = 2.176 \text{ A}$$

$$\theta_2 = \tan^{-1} \left(\frac{3 \times 2\pi \times 50 \times 20 \times 10^{-3}}{5} \right)$$

$$= 75.144^\circ$$

Then,

Load current up to 3rd order,

$$i_o(t) = 15.86 \sin(314.159t - 51.49^\circ) \\ + 2.176 \sin(942.48t - 75.144^\circ)$$

Then,

Harmonic Factor for the n^{th} harmonic is,

$$HF_n = \frac{V_n}{V_1}$$

$$\text{or, } HF_3 = \frac{V_3}{V_1} = \frac{42.44}{127.33} = 0.3333 \\ = 33.33\%$$

Now,

for Total Harmonic Distortion (THD),

$$THD = \frac{\sqrt{V_{0,rms}^2 - V_{1,rms}^2}}{V_{1,rms}}$$

Here,

$$V_{0,rms} = 100 \text{ V}$$

also,

$$V_{1, \text{rms}} = \frac{V_1 (\text{peak})}{\sqrt{2}}$$

$$= \frac{127.33}{\sqrt{2}}$$

$$= 90.04 \text{ V}$$

then,

$$\text{THD} = \frac{\sqrt{100^2 - 90.04^2}}{90.04}$$

$$= 0.4832$$

$$= 48.32\%$$

2078 Bhadra

Q. A single phase full bridge inverter has an RLC load with $R = 10 \Omega$, $L = 31.5 \text{ mH}$ and $C = 112 \mu\text{F}$. The inverter frequency is 50 Hz , and DC input voltage is 220 V .

- i) Express the instantaneous load current in fourier series
- ii) RMS value of fundamental component of load current
- iii) THD of load current

given,

$$V = 220 \text{ V}$$

$$R = 10 \Omega$$

$$L = 31.5 \text{ mH}$$

$$C = 112 \mu\text{F}$$

$$f = 50 \text{ Hz}$$

We have,

$$X_L = 2\pi fL$$

$$= 2\pi \times 50 \times 31.5 \times 10^{-3}$$

$$= 9.896 \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 112 \times 10^{-6}}$$

$$= 28.421 \Omega$$

then

$$Z_n = \sqrt{R^2 + \left(nX_L - \frac{X_C}{n}\right)^2}$$

~~the~~

$$\text{or, } Z_n = \sqrt{10^2 + \left(9.896n - \frac{28.421}{n}\right)^2}$$

then,

$$Z_1 = 21.052 \Omega$$

$$Z_3 = 22.553 \Omega$$

$$Z_n = R + j\left(nX_L - \frac{X_C}{n}\right)$$

$$\therefore Z_n = 10 + j\left(9.896n - \frac{28.421}{n}\right)$$

then,

$$Z_1 = 21.052 \angle +81.64^\circ \Omega$$

$$Z_3 = 22.553 \angle 63.678^\circ \Omega$$

$$Z_5 = 44.93 \angle 77.183^\circ$$

$$Z_7 = 65.974 \angle 81.281^\circ$$

$$Z_9 = \cancel{65.974} \angle \cancel{81.282}^\circ = 86.486 \angle 83.31^\circ$$

Then,

$$v_o(t) = \frac{4V}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin(n\omega t)$$

$$= \frac{4 \times 220}{\pi} \left[\sin(\omega t) + \frac{1}{3} \sin(3\omega t) \right.$$

$$+ \frac{1}{5} \sin(5\omega t) +$$

$$\frac{1}{7} \sin(7\omega t) +$$

$$\left. \frac{1}{9} \sin(9\omega t) \right]$$

$$= 280.113 \left[\sin(314.159t) + \frac{1}{3} \sin(3 \times 314.159t) \right.$$

$$+ \frac{1}{5} \sin(5 \times 314.159t) +$$

$$+ \frac{1}{7} \sin(7 \times 314.159t) +$$

$$\left. \frac{1}{9} \sin(9 \times 314.159t) \right]$$

$$v_o(t) = 280.113 \sin(314.159t) + 93.371 \sin(3 \times 314.159t)$$

$$+ 56.023 \sin(5 \times 314.159t) + 40.016 \sin(7 \times 314.159t)$$

$$+ 31.124 \sin(9 \times 314.159t)$$

Then,

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$$\begin{aligned}
 i_0(t) &= \frac{280.113}{21.052} \sin(314.159t + 61.64) \\
 &\quad + \frac{93.371}{22.553} \sin(3 \times 314.159t - 63.678) \\
 &\quad + \frac{56.023}{44.93} \sin(5 \times 314.159t + 77.183) \\
 &\quad + \frac{40.016}{65.974} \sin(7 \times 314.159t - 81.281) \\
 &\quad + \frac{31.124}{86.486} \sin(9 \times 314.159t + 83.36) \\
 &= 13.306 \sin(314.159t + 61.64) + 4.14 \sin(3 \times 314.159t - 63.678) \\
 &\quad + 1.247 \sin(5 \times 314.159t + 77.183) + 0.607 \sin(7 \times 314.159t - 81.281) \\
 &\quad + 0.36 \sin(9 \times 314.159t + 83.36)
 \end{aligned}$$

Then,

Peak value of load current,

$$\begin{aligned}
 &= \sqrt{13.306^2 + 4.14^2 + 1.247^2 + 0.607^2 + 0.36^2} \\
 &= 14.0086
 \end{aligned}$$

Then,

$$THD =$$

RMS value of fundamental component,

$$I_{rms,1} = \frac{13.306}{\sqrt{2}}$$

$$= 9.4088 \text{ A}$$

Then,

$$THD = \frac{\sqrt{I_{peak}^2 - I_{1,peak}^2}}{I_{1,peak}}$$

$$I_{1,peak}$$

$$= \frac{\sqrt{14.0086^2 - 13.306^2}}{13.306}$$

$$13.306$$

$$= 0.3292$$

$$= 32.923\%$$

A single phase full wave inverter has a resistive load of 5Ω . The DC input voltage is 30V.

Find,

- i) RMS value of o/p voltage
- ii) RMS value of fundamental component of o/p voltage
- iii) Output power
- iv) peak current in each thyristors.

Solⁿ,

i) For a square wave o/p having magnitude V_{dc} ,

$$V_o(\text{rms}) = V_{dc}$$

$$\therefore V_o(\text{rms}) = 30\text{V}$$

ii)

We have,

$$V_o(t) = \frac{4V}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin(n\omega t)$$

For fundamental component,

$$V_{\text{peak},1} = \frac{4 \times 30}{\pi} = 38.197\text{V}$$

then,

$$V_{\text{rms},1} = \frac{38.197}{\sqrt{2}} = 27.01\text{V}$$

A single phase full wave inverter has a resistive load of 5Ω . The DC input voltage is 30V.

Find,

- i) RMS value of o/p voltage
- ii) RMS value of fundamental component of o/p voltage
- iii) Output power
- iv) peak current in each thyristors.

Solⁿ,

- i) For a square wave o/p having magnitude V_{dc} ,

$$V_o(\text{rms}) = V_{dc}$$

$$\therefore V_o(\text{rms}) = 30\text{V}$$

- ii) We have,

$$V_o(t) = \frac{4V}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin(n\omega t)$$

For fundamental component,

$$V_{\text{peak},1} = \frac{4 \times 30}{\pi} = 38.197\text{V}$$

then,

$$V_{\text{rms},1} = \frac{38.197}{\sqrt{2}} = 27.01\text{V}$$

iii) $i_{o,rms}$ Output power,

$$P_o = \frac{V_o(rms)^2}{R}$$

$$= \frac{30^2}{5}$$

$$= 180 \text{ W}$$

iv)

For a resistive load,

$$i_o(t) = \frac{V_o(t)}{R}$$

i.e.

For peak,

~~$$I_{peak} = \frac{V_{peak}}{R}$$~~

for peak load current

$$I = \frac{V_{dc}}{R} = \frac{30}{5} = 6 \text{ A}$$

In a single phase full bridge inverter, two

Thyristors conduct at a time, and the current through each thyristor is equal to the load current.

$$\therefore I_{T, \text{peak}} = 6 \text{ A}$$

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Ex 4.11

given,

$$\text{Battery voltage} = 400 \text{ V}$$

$$R = 10 \Omega / \text{phase}$$

Star connected

120° mode of conduction

In 120° mode of conduction, at any instant, only two thyristors conduct (one phase floats), so two phases are in series across the source

Step 1

Two phase in series :

$$R_{eq} = R + R = 2R = 2 \times 10 = 20 \Omega$$

Peak value of load current,

$$I = \frac{V_{dc}}{R_{eq}} = \frac{400}{20} = 20 \text{ A}$$

Step 2

Each phase carries current for two-third of a cycle.

$$I_L(\text{rms}) = I \sqrt{\frac{2}{3}}$$

$$= 20 \times \sqrt{\frac{2}{3}}$$

$$= 16.33 \text{ A}$$

Step 3

$$\text{Power o/p} = 3 \times I_L(\text{rms})^2 \times R$$

$$= 3 \times 16.33^2 \times 10$$

$$= 8000.067 \text{ W}$$

Step 4

Each thyristor carries current for one-third of a cycle.

So,

$$I_T(\text{avg}) = \frac{I}{3} = \frac{20}{3} = 6.67 \text{ A}$$

Step 5

RMS thyristor current,

$$I_T(\text{rms}) = I \sqrt{\frac{1}{3}}$$

$$= 20 \times \sqrt{\frac{1}{3}}$$

$$= 11.55 \text{ A}$$

Step 6

Peak thyristor current,

Since only one thyristor conducts the full instantaneous current at any time in this mode:

$$I_T(\text{peak}) = I = 20 \text{ A}$$

2) For 180° Mode of conduction:

At any instant three thyristors conduct simultaneously. So, two phases are ~~connected~~ connected in parallel with each other, and this combination is in series with the third phase,

$$R_{eq} = R + \frac{R}{2} = 1.5R$$

$$= 1.5 \times 10$$

$$= 15 \Omega$$

Step 2

Instantaneous (peak) load current,

$$I_1 = I_2 = I_3 = \frac{V_{dc}}{R_{eq}}$$

$$= \frac{400}{15}$$

$$= 26.67 \text{ A}$$

Step 3

For RMS load current,

Each phase gets full current for one-third cycle and half the current for other two third cycle,

$$I_L(\text{rms}) = \sqrt{\frac{1}{2\pi} \left[\int_0^{2\pi/3} 26.67^2 d\omega + \int_{2\pi/3}^{2\pi} 13.33^2 d\omega \right]}$$

$$= \sqrt{\frac{1}{2\pi} \left[26.67^2 \left[\frac{2\pi}{3} \right] + 13.33^2 \left[\frac{4\pi}{3} \right] \right]}$$

$$= 18.856 \text{ A}$$

Step 4

$$\begin{aligned}
 \text{Power o/p} &= 3 I_{L \text{ rms}}^2 R \\
 &= 3 \times 18.856^2 \times 10 \\
 &= 10.66 \text{ kW}
 \end{aligned}$$

Each thyristor carries 26.67 A for a period of 60° (one-sixth of a cycle), 13.33 A for a period of 120° (i.e. one-third of cycle) and zero current for 180° .

Then,

Average current of thyristor,

$$= \frac{1}{2\pi} \left[\int_0^{\pi/3} 26.67 d\theta + \int_{\pi/3}^{\pi} 13.33 d\theta \right]$$

$$= \frac{1}{2\pi} \left[26.67 \times \frac{\pi}{3} + 13.33 \times \frac{2\pi}{3} \right]$$

$$= 8.89 \text{ A}$$

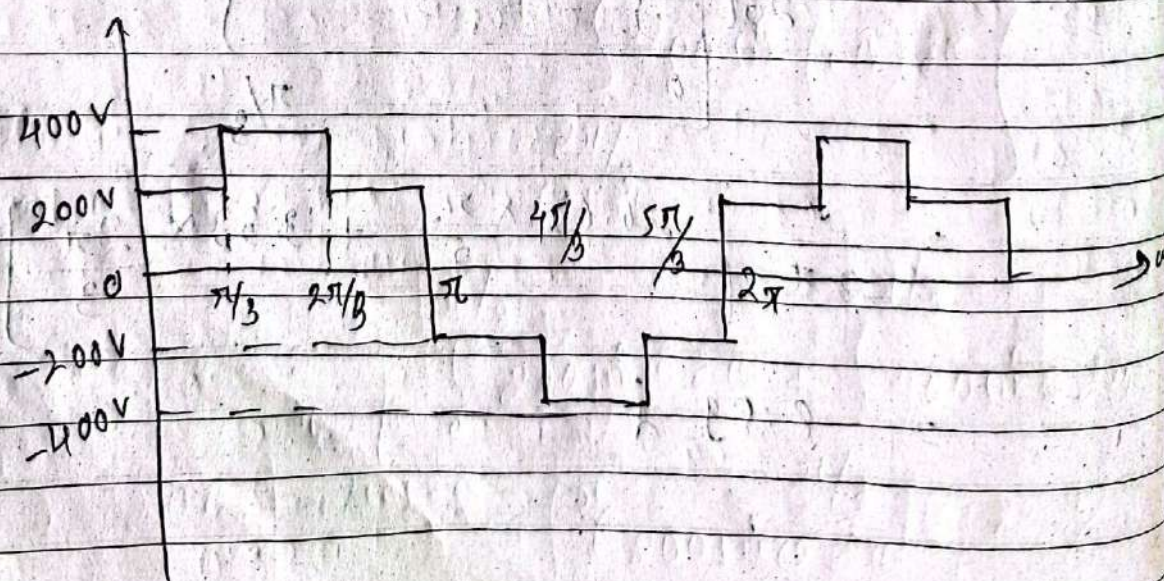
RMS Value of thyristor current,

$$= \sqrt{\frac{1}{2\pi} \left[\int_0^{\pi/3} 26.67^2 d\omega + \int_{\pi/3}^{\pi} 13.33^2 d\omega \right]}$$

$$= 13.33 \text{ A}$$

2029 Bhadra

Calculate the RMS value of fundamental and 3rd harmonics component of o/p voltage for 180° conduction mode of three phase inverter shown in figure below



Here,

From the figure

$$\frac{V_{dc}}{3} = 200V$$

then,

$$V_{dc} = 600V$$

and,

$$\frac{2V_{dc}}{3} = 400V$$

gives,

$$V_{an} = \begin{cases} 200, & 0 < \omega t < \pi/3 \\ 400, & \pi/3 < \omega t < 2\pi/3 \\ 200, & 2\pi/3 < \omega t < \pi \end{cases}$$

Let $\alpha = \omega t$

The fourier series is,

$$V_{an} = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\alpha + b_n \sin n\alpha)$$

Due to the symmetry of the waveform,

$$a_0 = 0$$

$$a_n = 0$$

Hence,

$$V_{an} = \sum_{n=1,3,5,\dots} b_n \sin(n\omega)$$

where,

$$b = \frac{2}{\pi} \int_0^{\pi} V_{an} \sin(n\omega) d\omega$$

Because the positive half cycle is also symmetric about

We have

$$b = \frac{2}{\pi} \left[\int_0^{\pi/3} 200 \sin(n\omega) d\omega + \int_{\pi/3}^{2\pi/3} 400 \sin(n\omega) d\omega + \int_{2\pi/3}^{\pi} 200 \sin(n\omega) d\omega \right]$$

For $n=1$,

$$b_1 = \frac{2}{\pi} \left[\int_0^{\pi/3} 200 \sin \omega d\omega + \int_{\pi/3}^{2\pi/3} 400 \sin \omega d\omega + \int_{2\pi/3}^{\pi} 200 \sin \omega d\omega \right]$$

$$= \frac{2}{\pi} \left[200 \left[-\cos 0 \right]_0^{\pi/3} + 400 \left[-\cos 0 \right]_{\pi/3}^{2\pi/3} + 200 \left[-\cos 0 \right]_{2\pi/3}^{\pi} \right]$$

$$= \frac{2}{\pi} \left[-200 \left[\cos \frac{\pi}{3} - \cos 0 \right] - 400 \left[\cos \frac{2\pi}{3} - \cos \frac{\pi}{3} \right] - 200 \left[\cos \pi - \cos \frac{2\pi}{3} \right] \right]$$

$$= \frac{2}{\pi} \left[-200 \left(\frac{1}{2} - 1 \right) - 400 (\cos 120^\circ - \cos 60^\circ) \right]$$

$$= 200 \left[-1 - (-\frac{1}{2}) \right]$$

$$= \frac{2}{\pi} \left[100 + 400 + 100 \right]$$

$$= \frac{2}{\pi} \left[600 \right]$$

$$= 381.972 \text{ V}$$

Here,

$$b_1 = 381.972 \text{ V}$$

This is the peak fundamental voltage,

$$V_1 (\text{rms}) = \frac{381.972}{\sqrt{2}}$$

$$= 270.09 \text{ V}$$

=) Third harmonic, $n=3$

$$b_3 = \frac{2}{\pi} \left[200 \int_0^{\pi/3} \sin 3\theta d\theta + 400 \int_{\pi/3}^{2\pi/3} \sin 3\theta d\theta + 200 \int_{2\pi/3}^{\pi} \sin 3\theta d\theta \right]$$

Since,

$$\int \sin 3\theta d\theta = -\frac{\cos 3\theta}{3}$$

We evaluate in 3 parts,

$$\rightarrow 200 \left[-\frac{\cos 3\theta}{3} \right]_0^{\pi/3} = -\frac{200}{3} \left[\cos 3 \times \frac{\pi}{3} - \cos 0 \right]$$

$$= -\frac{200}{3} [-1 - 1]$$

$$= -\frac{200}{3} (-2)$$

$$= \frac{400}{3}$$

$$\rightarrow 400 \left[-\frac{\cos 3\theta}{3} \right]_{\pi/3}^{2\pi/3} = -\frac{400}{3} \left[\cos 3 \times \frac{2\pi}{3} - \cos 3 \times \frac{\pi}{3} \right]$$

$$= -\frac{400}{3} [\cos 2\pi - \cos \pi]$$

$$= -\frac{400}{3} [1 - (-1)]$$

$$= -\frac{400}{3} [2]$$

$$= -\frac{800}{3}$$

$$\rightarrow 200 \left[-\frac{\cos 3\theta}{3} \right]_{2\pi/3}^{\pi} = -\frac{200}{3} [\cos 3\pi - \cos 3 \times \frac{2\pi}{3}]$$

$$= -\frac{200}{3} [-1 - 1]$$

$$= \frac{400}{3}$$

then,

$$b_g = \frac{2}{\pi} \left[\frac{400}{3} - \frac{800}{3} + \frac{400}{3} \right]$$

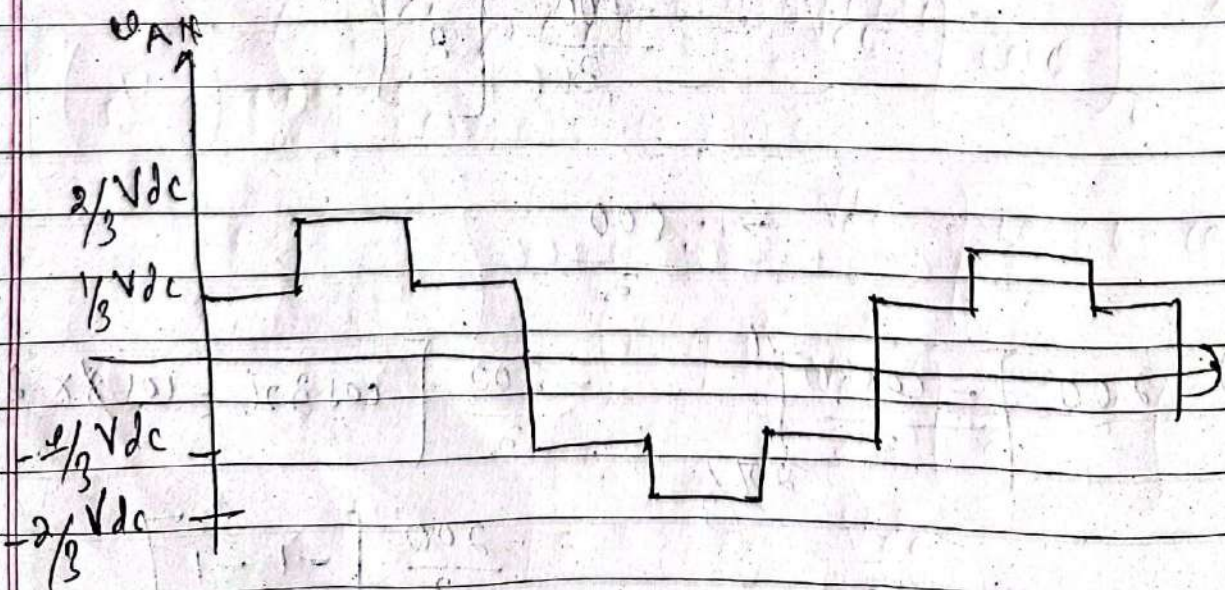
$$= 0$$

i.e.

$$V_g(\text{rms}) = 0V$$

2024 Chaitra

The given waveform in figure below is the o/p voltage of three phase inverter. Calculate the fundamental component and 3rd harmonic component of the o/p voltage



Here, from the figure,

$$V_{AN} = \begin{cases} \frac{V_{dc}}{3}, & 0 \leq \alpha \leq \pi/3 \\ \frac{2V_{dc}}{3}, & \pi/3 \leq \alpha \leq 2\pi/3 \\ \frac{V_{dc}}{3}, & 2\pi/3 \leq \alpha \leq \pi \end{cases}$$

Let, $\theta = \omega t$

The Fourier series is,

$$V_{AN} = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta)$$

Due to the symmetry of the waveform,

$$a_0 = 0$$

$$a_n = 0$$

Hence,

$$V_{an} = \sum_{n=1,3,5} b_n \sin(n\theta)$$

where,

Using half wave symmetry,

$$b_n = \frac{2}{\pi} \int_0^{\pi} v_{AN} \sin(n\omega) d\omega$$

For the fundamental component $n = 1$,

$$b_1 = \frac{2}{\pi} \left[\int_0^{\pi/3} \frac{V_{dc}}{3} \sin \theta d\theta + \int_{\pi/3}^{2\pi/3} \frac{2V_{dc}}{3} \sin \theta d\theta + \int_{2\pi/3}^{\pi} \frac{V_{dc}}{2} \sin \theta d\theta \right]$$

Evaluate each integral:

$$\int_0^{\pi/3} \frac{V_{dc}}{3} \sin \theta d\theta = -\frac{V_{dc}}{3} \left[\cos \theta \right]_0^{\pi/3}$$

$$= -\frac{V_{dc}}{3} [\cos 60^\circ - \cos 0^\circ]$$

$$= -\frac{V_{dc}}{3} \left[\frac{1}{2} - 1 \right]$$

$$= \frac{V_{dc}}{6}$$

$$\begin{aligned}
 & \int_{\pi/3}^{2\pi/3} \frac{2V_{dc}}{3} \sin \alpha d\alpha = \frac{2V_{dc}}{3} [\cos \alpha]_{\pi/3}^{2\pi/3} \\
 & = \frac{-2V_{dc}}{3} \left[\cos \frac{2\pi}{3} - \cos \frac{\pi}{3} \right] \\
 & = \frac{-2V_{dc}}{3} [-1] \\
 & = \frac{2V_{dc}}{3}
 \end{aligned}$$

$$\begin{aligned}
 & \int_{2\pi/3}^{\pi} \frac{V_{dc}}{3} \sin \alpha d\alpha = -\frac{V_{dc}}{3} [\cos \alpha]_{2\pi/3}^{\pi} \\
 & = -\frac{V_{dc}}{3} \left[\cos \pi - \cos \frac{2\pi}{3} \right] \\
 & = -\frac{V_{dc}}{3} \left[-1 + \frac{1}{2} \right] \\
 & = \frac{V_{dc}}{6}
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 b_1 &= \frac{2}{\pi} \left[\frac{V_{dc}}{6} + \frac{2V_{dc}}{3} + \frac{V_{dc}}{6} \right] \\
 &= \frac{2}{\pi} V_{dc}
 \end{aligned}$$

Hence,

the fundamental component is,

$$V_{AN,1} = \frac{2V_{dc}}{\pi} \sin \omega t$$

its peak,

$$\begin{aligned} V_{AN,1, \text{peak}} &= \frac{2V_{dc}}{\pi} \\ &= 0.6366 V_{dc} \end{aligned}$$

its rms value,

$$\begin{aligned} V_{AN,1(\text{rms})} &= \frac{2V_{dc}}{\pi \sqrt{2}} \\ &= 0.4502 V_{dc} \end{aligned}$$

=) For third harmonic component,

$$n = 3,$$

$$b_3 = \frac{2}{\pi} \int_0^{\pi} V_{AN} \sin 3\omega t d\omega$$

Therefore,

$$b_g = \frac{2}{\pi} \left[\int_0^{\pi/3} \frac{V_{dc}}{3} \sin 3\theta d\theta + \int_{\pi/3}^{2\pi/3} \frac{2V_{dc}}{3} \sin 3\theta d\theta + \int_{2\pi/3}^{\pi} \frac{V_{dc}}{3} \sin 3\theta d\theta \right]$$

The three terms are,

$$\frac{2V_{dc}}{9}, \quad -\frac{4V_{dc}}{9}, \quad \frac{2V_{dc}}{9}$$

then,

$$b_g = \frac{2}{\pi} \left[\frac{2V_{dc}}{9} - \frac{4V_{dc}}{9} + \frac{2V_{dc}}{9} \right]$$

$$\therefore b_g = 0$$

$$\therefore U_{AN3} = 0$$

AC Voltage Controller

22

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207 | Shrawan

A single-phase full wave AC voltage controller feeds a load of resistance 20Ω with an input voltage of $230V, 50Hz$. If the firing angle for both thyristors is 45° , calculate

- i) RMS value of o/p voltage
- ii) Load power and i/p power factors
- iii) Average and rms current of thyristors

Given,

$$R = 20\Omega$$

$$V = 230V, 50Hz$$

$$\alpha = 45^\circ = \frac{\pi}{4}$$

i) RMS o/p voltage across load,

$$V_o = V_m \sqrt{\frac{1}{2\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

(rms)

where,

$$V_m = \sqrt{2} \times V$$

$$= \sqrt{2} \times 230$$

$$= 325.27V$$

then,

$$V_o(\text{rms}) = 325.27 \sqrt{\frac{1}{2\pi} \left[\left(\pi - \frac{\pi}{4} \right) + \frac{\sin(2 \times \pi/4)}{2} \right]}$$

$$\Rightarrow 219.31 \text{ V}$$

then,

Load Power,

$$P_o = \frac{V_o(\text{rms})^2}{R}$$

$$= \frac{219.31^2}{20}$$

$$= 2404.844 \text{ Watt}$$

$$= 2.405 \text{ kW}$$

then,

$$I_o(\text{rms}) = \frac{V_o(\text{rms})}{R} = \frac{219.31}{20}$$

$$= 10.966 \text{ A}$$

then,

$$PF = \frac{P_o(\text{rms})}{V_o(\text{rms}) \times I_o(\text{rms})} = \frac{2404.844}{230 \times 10.966}$$

$$= 0.9535 \text{ lag}$$

* Note,

$$I_T(\text{avg}) = \frac{V_m}{2\pi R} [1 + \cos \alpha]$$

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Now

Average thyristor current

$$= \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m \sin \omega t}{R} d(\omega t)$$

$$= \frac{V_m}{2\pi R} \left[\cos \omega t \right]_{\pi/4}^{\pi}$$

$$= \frac{V_m}{2\pi R} \left[\cos \pi - \cos \frac{\pi}{4} \right]$$

$$= \frac{V_m}{2\pi R} \left[-1 - \frac{1}{\sqrt{2}} \right]$$

$$= \frac{V_m}{2\pi R} [1.707]$$

$$= \frac{325.27}{2\pi \times 20} [1.707]$$

$$= 4.418 \text{ A}$$

RMS thyristor current

Each thyristor carries the current for 50% of the total time. So,

$$I_T(\text{rms}) = \frac{I_o(\text{rms})}{\sqrt{2}}$$

$$= \frac{10.966}{\sqrt{2}}$$

$$= 7.754 \text{ A}$$

2073 Chaitra

A single phase Full wave AC voltage controller feeds a load of $R = 10 \Omega$ with an AC i/p voltage of 230 V , 50 Hz . Firing angle for both thyristors is 45° . Calculate rms value of o/p voltage, load current, input power factor and average value of current of thyristor.

Here, given,

$$R = 10 \Omega$$

$$\alpha = 45^\circ = \frac{\pi}{4}$$

$$V_i = V_s = 230 \text{ V}$$

Then,

$$\begin{aligned}
 V_o(\text{rms}) &= V_s \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]} \\
 &= 230 \sqrt{\frac{1}{\pi} \left[\left(\pi - \frac{\pi}{4} \right) + \frac{\sin(2 \times \pi/4)}{2} \right]} \\
 &= 219.304 \text{ V}
 \end{aligned}$$

Then,

$$\begin{aligned}
 I_o(\text{rms}) &= \frac{V_o(\text{rms})}{R} \\
 &= 219.304
 \end{aligned}$$

$$= 21.930 \text{ A}$$

Then,

$$\begin{aligned}
 P_o(\text{rms}) &= \frac{V_o(\text{rms})^2}{R} \\
 &= \frac{219.304^2}{10}
 \end{aligned}$$

$$= 4809.424 \text{ W}$$

Then,

$$\text{pf} = \frac{P_o, \text{rms}}{V_s I_o, \text{rms}}$$

$$= \frac{4809.424}{230 \times 21.930}$$

$$= 0.954 \text{ lagging}$$

also,

$$I_T (\text{avg}) = \frac{V_m}{2\pi R} [1 + \cos \alpha]$$

$$= \frac{\sqrt{2} \times 230}{2\pi \times 10} \left[1 + \cos \left(\frac{\pi}{4} \right) \right]$$

$$= 8.8374 \text{ A}$$

The single phase AC controller has input voltage of 230V, 50Hz and the positive and negative thyristors are triggered at an angle of 90° and $\pi + 90^\circ$ respectively. The series RL load has $R = 20\Omega$, $L = 50mH$, then determine the RMS value of load voltage and load current given,

$$V = 230V \text{ (rms)}$$

$$f = 50Hz$$

$$R = 20\Omega$$

$$L = 50mH = 50 \times 10^{-3}H$$

positive thyristor firing angle,

$$\alpha = 90^\circ = \frac{\pi}{2}$$

Now, for $V_{o,rms}$ and $I_{o,rms}$,

(Step)

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 50$$

$$\therefore \omega = 314.16 \text{ rad/sec}$$

Step 2

Peak values

$$V_m = \sqrt{2} V$$

$$= \sqrt{2} \times 230$$

$$= 325.269 \text{ V}$$

Step 3

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{20^2 + (314.16 \times 50 \times 10^{-3})^2}$$

$$= 25.431 \Omega$$

Step 4load phase angle ϕ ,

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$= \tan^{-1} \left(\frac{314.16 \times 50 \times 10^{-3}}{20} \right)$$

$$\phi = 38.146^\circ$$

then,

$$\tan \phi = \tan(38.146^\circ)$$

$$= 0.7854$$

Now, for extinction angle β ,

We have, for the series RL load,

$$i_o = \frac{\sqrt{2}V}{2} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{\alpha - \omega t}{\tan \phi}} \right]$$

At the extinction angle, β , current becomes zero.

$$i_o(\beta) = 0$$

$$\text{or, } \sin(\beta - \phi) - \sin(\alpha - \phi) e^{\frac{\alpha - \beta}{\tan \phi}} = 0$$

$$\text{or, } \sin(\beta - \phi) = \sin(\alpha - \phi) e^{\frac{\alpha - \beta}{\tan \phi}}$$

$$\text{or, } \sin(\beta - 38.146) = \sin(90 - 38.146) e^{\frac{90 - \beta}{0.7854}}$$

$$\text{or, } \sin(\beta - 38.146) = \sin(51.854) e^{\frac{90 - \beta}{0.7854}}$$

Using radian values,

$$\sin(\beta - 0.6658) = \sin(0.9050) e^{\frac{1.571 - \beta}{0.7854}}$$

Since the current continues beyond 180° ,

$$\pi < \beta < 2\pi$$

i.e.

$$3.1416 < \beta < 6.2832$$

Solving

$$\beta = 3.7588 \text{ rad}$$

$$= 215.363^\circ$$

Then,

$$\text{Conduction angle} = \beta - \alpha$$

$$= 3.7588 - 1.571$$

$$= 2.189 \text{ rad}$$

Then,

RMS value of load voltage,

$$V_{\text{orms}} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\beta} V_m^2 \sin^2 \theta d\theta}$$

$$= \sqrt{\frac{1}{\pi}} \int_{1.571}^{3.7588} V_m^2 \sin^2 \omega \, d\omega$$

$$= V_m \sqrt{\frac{1}{\pi}} \int_{1.571}^{3.7588} \left(\frac{1 - \cos 2\omega}{2} \right) d\omega$$

$$= V_m \sqrt{\frac{1}{\pi}} \left[\frac{\omega}{2} - \frac{\sin 2\omega}{4} \right]_{1.571}^{3.7588}$$

$$= V_m \sqrt{\frac{1}{\pi}} \left[\frac{1}{2} [3.7588 - 1.571] \right]$$

$$- \frac{1}{4} [\sin 2 \times 3.7588 - \sin 2 \times 1.571]$$

$$= 169.966$$

$$\approx 170 \text{ V}$$

then,

$$\text{For } I_{0, \text{rms}} = \frac{V_{0, \text{rms}}}{Z}$$

$$= \frac{170}{25.481}$$

again,

$$I_{o,rms} \neq \frac{V_{o,rms}}{Z}$$

(Here, the o/p is chopped waveform and the current contains an exponential transient term)

$$i_o(\alpha) = \frac{\sqrt{2} V_s}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{\alpha - \omega t}{\tan \phi}} \right]$$

$$= \frac{325.269}{25.431} \left[\sin(0 - \phi) - \sin(0.9050) e^{\frac{1.571 - 0}{0.7854}} \right]$$

Then,

$$I_{o,rms} = \sqrt{\frac{1}{\pi} \int_{1.571}^{3.7588} i_o^2(\alpha) d\alpha}$$

$$= 3.574 \text{ A}$$

A single phase AC voltage controller with 230V i/p voltage is connected to RL load $R = 2\Omega$ and $X_L = 2\Omega$ is operated at firing angle of 45° . The o/p of AC voltage controller is continuous.

- Calculate,
- Extinction angle
 - Conduction angle
 - RMS value of o/p voltage
 - Draw its waveform

Here, given,

$$V = 230V$$

$$R = 2\Omega$$

$$X_L = 2\Omega$$

$$\alpha = 45^\circ = \frac{\pi}{4}$$

then,

$$\omega = 2\pi f = 314.16 \text{ rad/sec}$$

also,

peak voltage,

$$V_m = V\sqrt{2} = 230 \times \sqrt{2} = 325.27V$$

then,

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{2^2 + (314.16 \times 2)^2}$$

$$= \sqrt{2^2 + 2^2}$$

$$= 2\sqrt{2} \Omega$$

$$= 2.828 \Omega$$

then,

Load phase angle,

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$= \tan^{-1} \left(\frac{314.16 \times 2}{2} \right)$$

$$= \tan^{-1} \left(\frac{2}{2} \right)$$

$$= 0.7854 \text{ rad}$$

then,

$$\tan \phi = \tan(0.7854)$$

$$= 1$$

Now, ~~for~~ for the series RL load,

$$i_0 = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{\alpha - \omega t}{\tan \phi}} \right]$$

At the extinction angle β ,

$$i_o(\beta) = 0$$

$$\text{or, } \sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{\alpha - \omega t}{\tan \phi}} = 0$$

$$\text{or, } \sin(\beta - 0.7854) - \sin(0.7854 - 0.7854) e^{\frac{\alpha - \beta}{\tan \phi}} = 0$$

$$\text{or, } \sin(\beta - 0.7854) = 0$$

$$\text{or, } \sin(\beta - 0.7854) = \sin(\pi)$$

$$\text{or, } \beta - 0.7854 = \pi$$

$$\therefore \beta = 3.927 \text{ rad}$$

Then,

$$\text{Conduction angle} = \beta - \alpha$$

$$= 3.927 - \frac{\pi}{4}$$

$$= 3.1416$$

Now,

Using the general RMS voltage formula for a single phase AC controller:

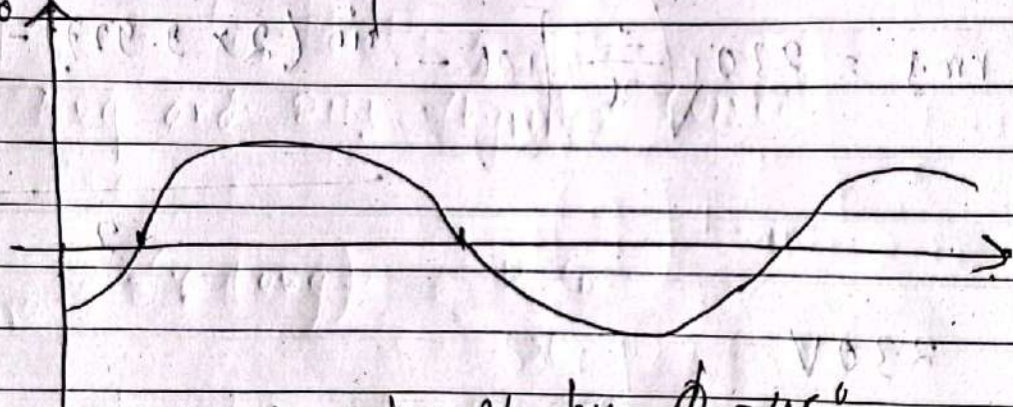
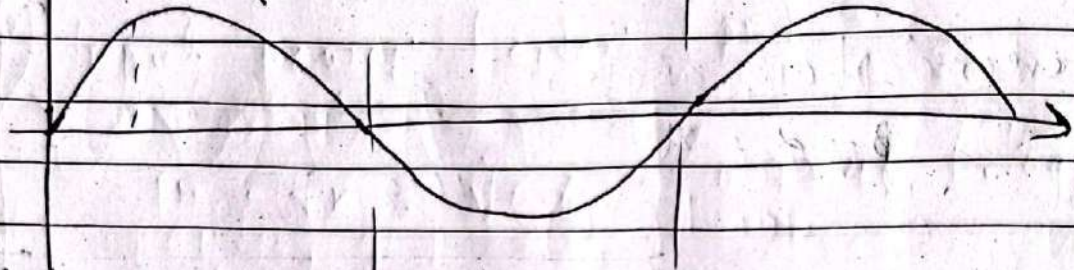
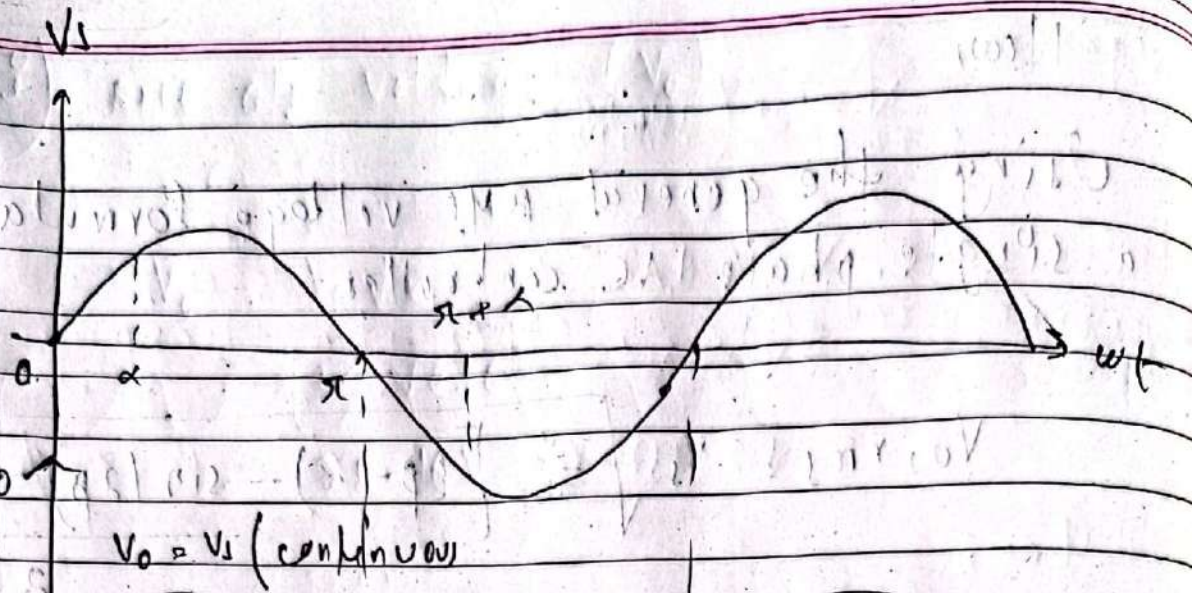
$$V_{o, rms} = V_s \sqrt{\frac{1}{\pi} \left[(\beta - \alpha) - \frac{\sin(2\beta) - \sin(2\alpha)}{2} \right]}$$

Here,

$$\beta - \alpha = \pi$$

$$V_{o, rms} = 230 \sqrt{\frac{1}{\pi} \left[\pi - \frac{\sin(2 \times 3.927) - \sin(2 \times 0.7854)}{2} \right]}$$

$$= 230V$$



Sinusoidal lagging V_0 by $\phi = 45^\circ$

A single phase - AC voltage control has input voltage of 230V, 50 Hz and a load of resistor of 15 Ω . For 6 cycles ON and 4 cycles OFF, calculate

i) RMS o/p voltage

ii) Input pf

iii) Average and RMS thyristor current

iv) Draw input and output wave form

Here,

$$\text{ON } (n) = 6$$

$$\text{OFF } (m) = 4$$

Then,

$$k = \frac{n}{n+m} = \frac{6}{6+4} = 0.6$$

given,

$$\text{RMS } (V_s) = 230 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$R = 15 \Omega$$

$$\text{Peak Input Voltage } (V_m) = \sqrt{2} \times 230$$

$$= 325.269 \text{ V}$$

$$\text{Peak load current } (I_m) = \frac{V_m}{R}$$

$$= \frac{325.296}{15} = 21.686 \text{ A}$$

ii) RMS o/p voltage (V_o),

$$V_o = V_s \sqrt{k}$$

$$= 230 \sqrt{0.6}$$

$$= 178.16 \text{ V}$$

iii) Input power factor (PF):

$$PF = \sqrt{k} = \sqrt{0.6} = 0.7746 \text{ (Lagging)}$$

iv) Avg and RMS Thyristor Current

$$I_T (\text{avg}) = \frac{k I_m}{\pi}$$

$$= \frac{0.6 \times 21.686}{\pi}$$

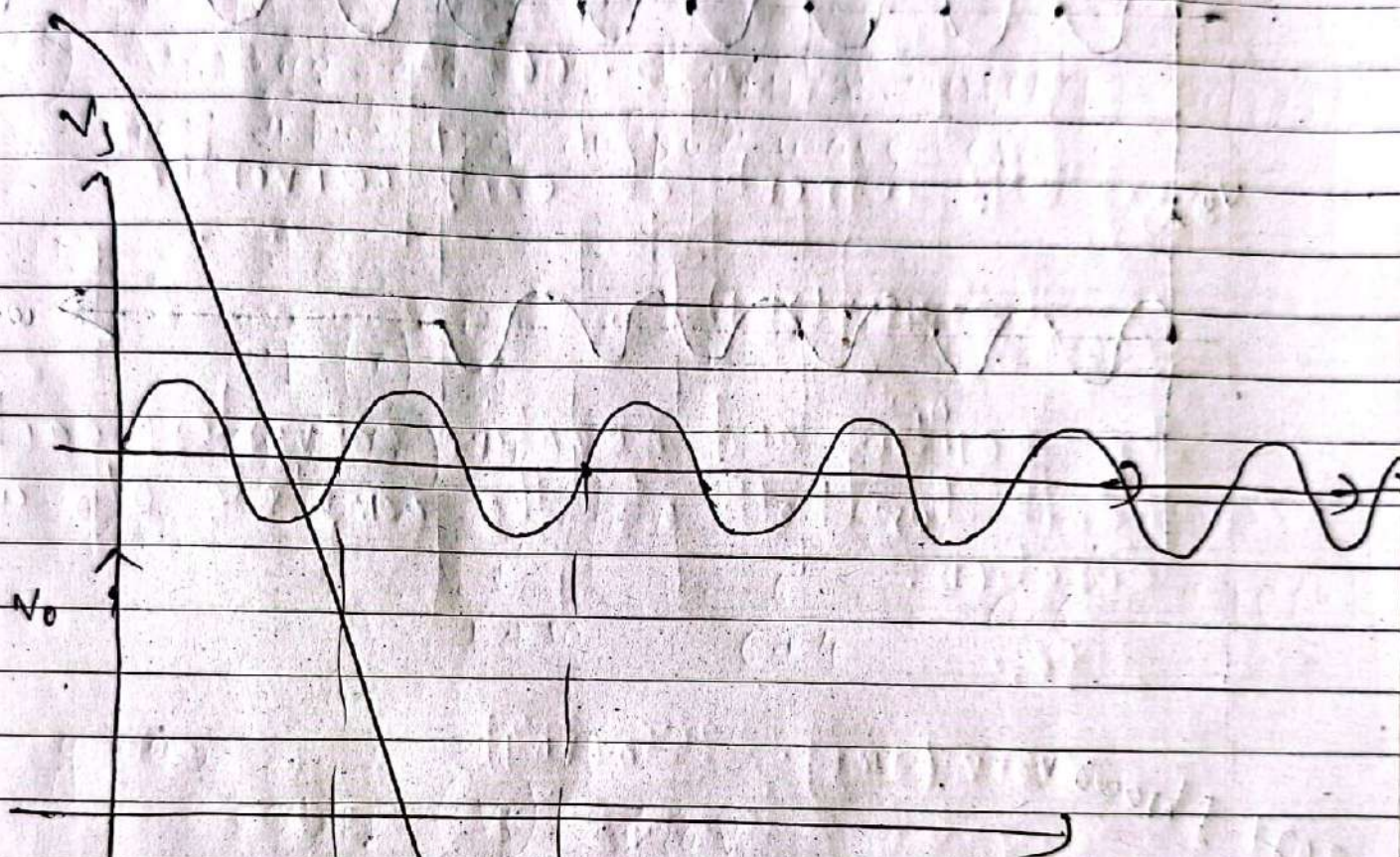
$$= 4.14 \text{ A}$$

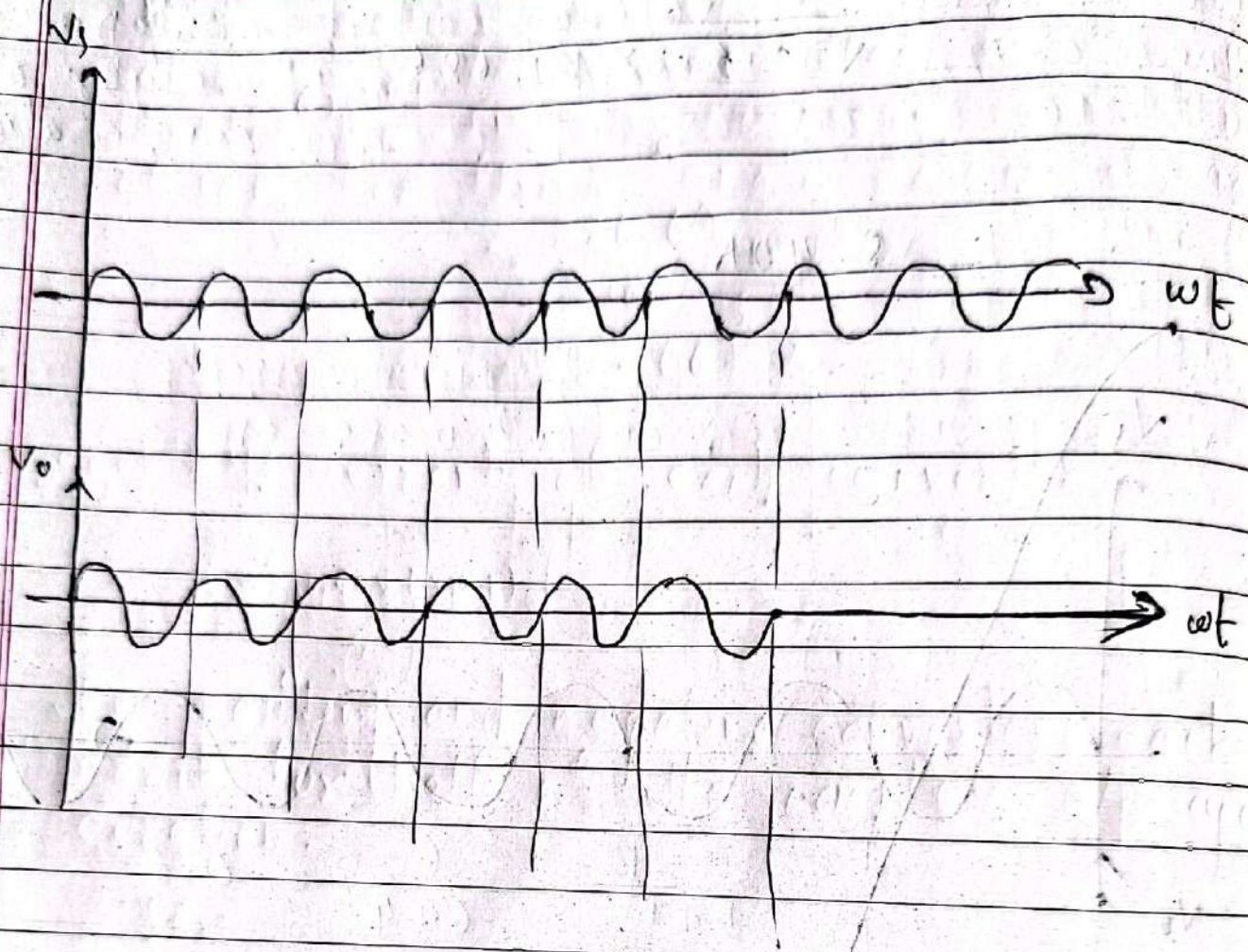
RMS Thyristor current,

$$I_T (\text{rms}) = \frac{\sqrt{k} I_m}{2}$$

$$= \frac{\sqrt{0.8} \times 21.68}{2}$$

$$= 8.40 \text{ A}$$





2071 Shown

A single phase bridge type cyclo-converter has input voltage of 230V , 50Hz and load of $R = 10\Omega$. The o/p frequency is one third input frequency. For firing angle of 30° ,

calculate

- 1) RMS ^{value} of o/p voltage
- 2) RMS value of o/p current
- 3) RMS current of each converter

Q) RMS current of each thyristor

Soln

Q) For a single phase cycloconverter, supplying a resistive load, each half cycle of input conducts from angle α to π .

given

$$V_s = 230 \text{ V}$$

$$f_s = 50 \text{ Hz}$$

$$f_o = \frac{f_s}{3} = 16.67 \text{ Hz}$$

$$R = 10 \Omega$$

$$\alpha = \frac{\pi}{6} = 30^\circ$$

Then,

$$V_o, \text{ rms} = V_s \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

$$= 230 \sqrt{\frac{1}{\pi} \left[\left(\pi - \frac{\pi}{6} \right) + \frac{\sin(2 \times \pi/6)}{2} \right]}$$

$$= 226.67 \text{ V}$$

ii)

$$I_o, \text{ rms} = \frac{V_o, \text{ rms}}{R}$$

$$= \frac{226.67}{10}$$

$$= 22.667 \text{ A}$$

iii) RMS current of each converter

$$[I_{\text{conv(rms)}}],$$

the positive and negative converters each conduct for half of the complete o/p cycle duration.

So,

$$I_{\text{conv(rms)}} = \frac{I_o(\text{rms})}{\sqrt{2}}$$

$$= \frac{22.667}{\sqrt{2}}$$

$$= 16.09 \text{ A}$$

iv)

In a full bridge confⁿ, each of the 4 thyristors in a converter conduct during alternating half-cycles, carrying current for half of that converter's operating duration.

$$I_r(\text{rms}) = I_{\text{conv}}(\text{rms})$$

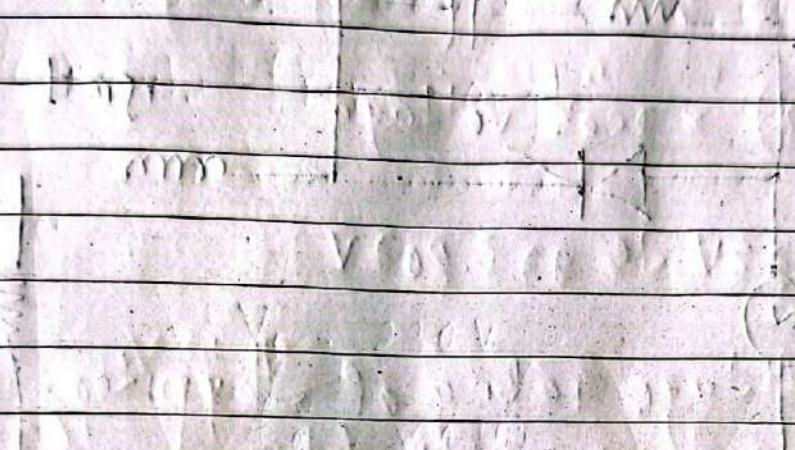
$$\sqrt{2}$$

$$I_o(\text{rms})$$

$$= \frac{16.03}{\sqrt{2}}$$

$$\sqrt{2}$$

$$= 11.33 \text{ A}$$

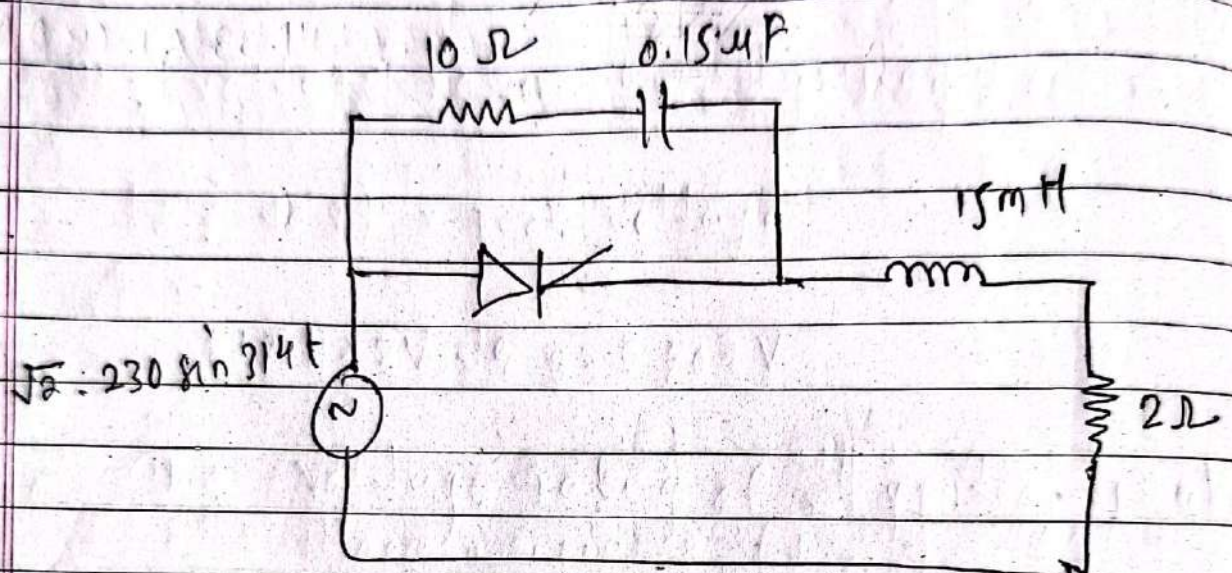


2075 Ashwin

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For the circuit shown is i) calculate the max value of di/dt and dv/dt of the SCR
ii) Find the RMS and average current rating of SCR for firing angle delays of 90°



given,

$$V_s = \sqrt{2} \cdot 230 \sin 314t$$

$$R_s = 10 \Omega$$

$$C_s = 0.15 \mu F$$

$$R = 2 \Omega$$

$$L = 15 mH = 15 \times 10^{-3} H = 15 \times 10^{-3} H$$

then,

$$\left. \frac{di}{dt} \right|_{max} = \frac{V_s}{L} = \frac{230 \sqrt{2}}{15 \times 10^{-3}}$$

$$= \frac{230 \sqrt{2}}{15 \times 10^{-3}}$$

$$= 21,684.61 A/s$$

$$= 0.021685 \text{ A/us}$$

then,

$$\left. \frac{dV}{dt} \right|_{\max} = R_s \left(\frac{dI_o}{dt} \right)_{\max}$$

$$= 10 \times 21,684.6 \times 0.021685$$

$$= 0.21685 \text{ V/us}$$

given,

$$\alpha = 90^\circ$$

then, the max^m load current I_L ,

$$I = \frac{V_s}{R} = \frac{\sqrt{2} \times 230}{102} = 162.634 \text{ A}$$

then,

at firing angle,

$$I_r(\text{avg}) = \frac{\sqrt{2}}{\pi} I_m$$

$$= \frac{\sqrt{2}}{\pi} \times 162.634$$

$$= 57.49 \text{ A}$$

The rms current rating of thyristor is

$$162.634 \text{ A}$$

Now,

The voltage rating of thyristor is,

$(2.5 \text{ to } 3) \times \text{peak voltage}$

$$= 813.173 \text{ V to } 975.807 \text{ V}$$

~~The voltage rating of thyristor is also~~

0081 Bhadra

A 10V source is connected across RL load having $R = 10 \Omega$ and $L = 10 \text{ mH}$. A thyristor having latching current of 200 mA is used as switch for the circuit. Calculate minimum gate pulse width to turn ON the thyristor.

Here, given,

$$V = 10 \text{ V}$$

$$I_L = 200 \text{ mA}$$

$$R = 10 \Omega$$

$$L = 10 \text{ mH} = 10 \times 10^{-3} \text{ H}$$

Then,

$$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}}{10} = 0.001$$

then,

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The current ~~is~~ rise in an RL ckt connected to a DC source V_s ,

$$i(t) = \frac{V_s}{R} \left(1 - e^{-t/\tau} \right)$$

$$\text{or, } 200 \times 10^{-3} = \frac{10}{10} \left(1 - e^{-t/0.001} \right)$$

$$\text{or, } 0.2 = 1 - e^{-t/0.001}$$

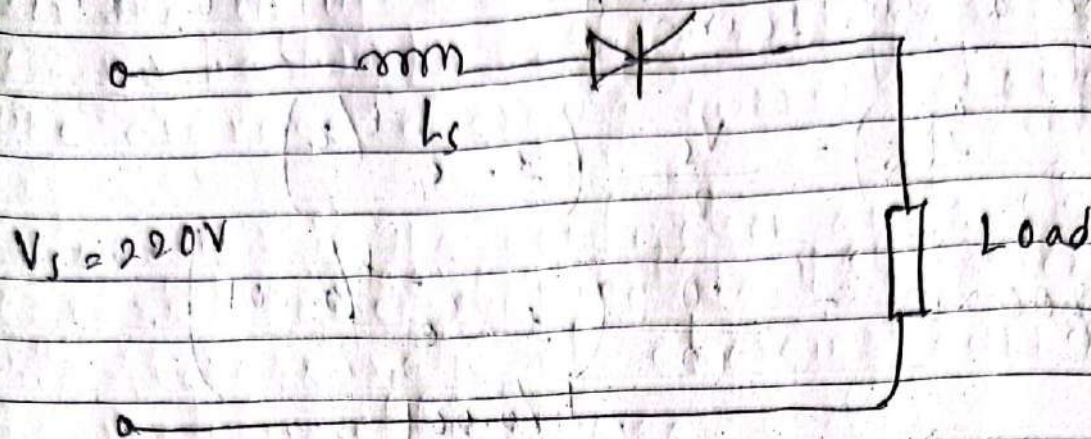
$$\text{or, } e^{-t/0.001} = 1 - 0.2 = 0.8$$

$$\text{or, } \frac{-t}{0.001} = \ln 0.8$$

$$t = 0.000223 \text{ sec} = 223.1 \mu\text{s}$$

2070 Asadh

Fig shows the single phase half-wave controlled rectifier with di/dt protection provided by the series inductor. The load is purely inductive having an inductance of 100mH . Calculate the value of L_s so that di/dt is limited to 500 A/sec ($V_s = 220\text{V}$).



given,

$$V_s = 220V$$

$$V_m = \sqrt{2} \times 220 = 311.13V$$

$$\text{Load inductance } (L_{\text{load}}) = 100mH$$

$$= 100 \times 10^{-3}$$

$$= 0.1H$$

given,

$$\text{Max}^n \frac{di}{dt} = 500A/s$$

We have,

$$\left(\frac{di}{dt} \right)_{\text{max}} = \frac{V_m}{L_{\text{total}}}$$

when,

$$L_{\text{total}} = L_s + L_{\text{load}}$$

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$$\text{or, } 500 = 811.13$$

$$L_{\text{total}}$$

$$\text{or, } L_{\text{total}} = 0.6223 \text{ H}$$

$$= 622.26 \text{ mH}$$

When,

$$L_s = L_{\text{total}} - L_{\text{load}}$$

$$\text{or } L_s = 622.26 - 100$$

$$= 522.26 \text{ mH}$$

2022
chairra

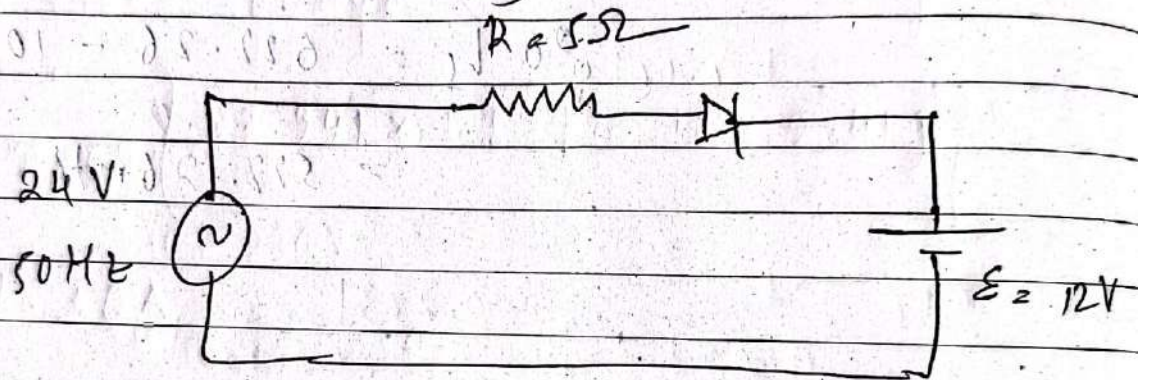
Single Phase AC to DC Conversion

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1.

For the circuit shown ($24V$, $50Hz$, $R = 5\Omega$, $\epsilon = 12V$ battery), battery voltage is $\epsilon = 12V$. The average charging current should be

Calculate: i) The conduction angle of the diode ii) Power supplied to the battery iii) Average value of charging current iv) The rectifier efficiency.



given,

$$V = 24V$$

$$I_{av} = 10A$$

$$\epsilon = 12V$$

$$\left\{ \begin{aligned} V_m &= \sqrt{2} \times 24 \\ &= 33.941V \end{aligned} \right.$$

a) The conduction angle α of the diode is,

$$\alpha = \sin^{-1} \left(\frac{\epsilon}{V_m} \right)$$

$$= \sin^{-1} \left(\frac{12}{33.941} \right)$$

$$= 0.361 \text{ rad}$$

$$= 20.71^\circ$$

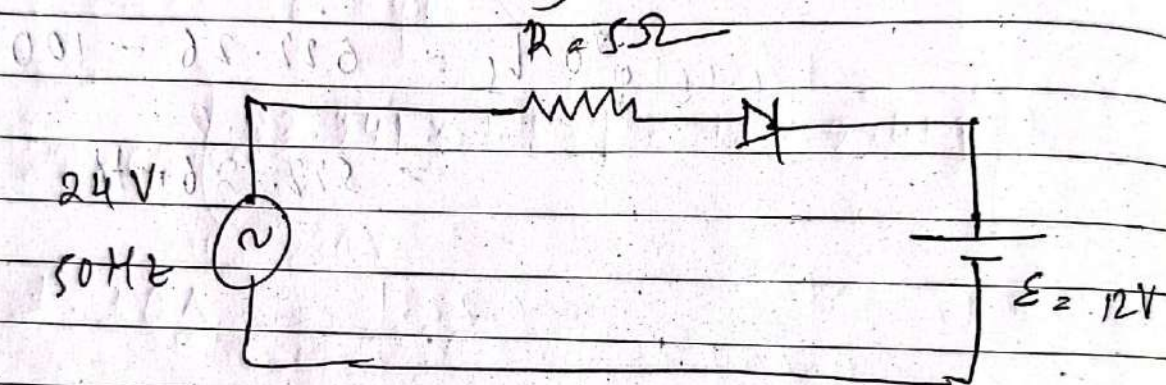
2022
Chaitra

Single Phase AC to DC Conversion

1.

For the circuit shown ($24V$, $50Hz$, $R = 5\Omega$, $E = 12V$ battery), battery voltage is $E = 12V$. The average charging current should be

Calculate, i) The conduction angle of the diode ii) Power supplied to the battery iii) Average value of charging current in the rectifier efficiency.



given,

$$V = 24V$$

$$I_{av} = 10A$$

$$E = 12V$$

$$\left. \begin{aligned} V_m &= \sqrt{2} \times 24 \\ &= 33.941V \end{aligned} \right\}$$

a) The conduction angle α of the diode is,

$$\alpha = \sin^{-1} \left(\frac{E}{V_m} \right)$$

$$= \sin^{-1} \left(\frac{12}{33.941} \right)$$

$$= 0.361 \text{ rad}$$

$$= 20.71^\circ$$

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i.e. conduction starts when supply voltage exceeds battery voltage, $V_m \sin \alpha = E$

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then, Conduction stops when supply falls back below E
 $\beta = \pi - \alpha$

$$= \pi - 0.361$$

$$= 159.29^\circ = 2.7801 \text{ rad}$$

then,

$$\text{Conduction angle } (\beta) = \beta - \alpha$$

$$= 159.29 - 20.71$$

$$= 138.58^\circ = 2.4187 \text{ rad}$$

iii) The avg value of charging current,

$$I_{\text{avg}} = \frac{1}{2\pi R} \int_{\alpha}^{\beta} (V_m \sin \theta - E) d\theta$$

$$= \frac{1}{2\pi R} \left[-V_m \cos \theta - E\theta \right]_{\alpha}^{\beta}$$

$$= \frac{1}{2\pi R} \left[33.941 \times [\cos 159.29 - \cos 20.71] \right]$$

$$- 12 [2.4187]$$

$$I_{rms} = \frac{1}{2\pi \times 5} \left[V_m (\cos \alpha - \cos \beta) - E (\beta - \alpha) \right]$$

$$= \frac{1}{2\pi \times 5} \left[83.941 (\cos(0.361) - \cos(2.780)) \right]$$

$$- 12 (2.780 - 0.361)$$

$$= \frac{1}{2\pi \times 5} \left[83.941 \times 1.871 - 12 \times 2.419 \right]$$

$$= 1.10 \text{ A}$$

ii) Power supplied to battery,

$$P_{\text{battery}} = E \times I_{avg}$$

$$= 12 \times 1.10$$

$$= 13.2 \text{ W}$$

iv) We have,

$$I_{rms}^2 = \frac{1}{2\pi} \int_{\alpha}^{\beta} \left(\frac{V_m \sin \omega t - E}{R} \right)^2 d\omega t$$

$$I_{rms}^2 = 3.8108$$

$$\therefore I_{rms} = 1.952$$

Now,

Total power supplied by the AC source (P_{ac}):

$$P_{ac} = P_{dc} + I_{rms}^2 R$$

$$= 13.2 + 1.952^2 \times 5$$

$$= 32.25 \text{ W}$$

Then,

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\%$$

$$= \frac{13.2}{32.25} \times 100\%$$

$$= 40.93\%$$

A diode whose internal resistance is 20Ω is to supply power to 1000Ω load from a 230 AC supply in case of half wave rectifier.

- Calculate,
i) Peak load current
ii) DC load current
iii) DC diode voltage

Here, given,

$$V_s = 230 \text{ V}$$

$$V_m = \sqrt{2} \times 230 \\ = 325.269 \text{ V}$$

$$r_d = 20 \Omega$$

$$R = 1000 \Omega$$

then,

- i) Peak load current,

$$I_m = \frac{V_m}{r_d + R_L} \\ = \frac{325.269}{20 + 1000} = 0.3189 \text{ A}$$

ii) DC Load Current (I_d)

For a half wave rectifier,

$$I_{dc} = \frac{I_m}{\pi} = \frac{0.3189}{\pi} = 0.1015 \text{ A} \\ = 101.51 \text{ mA}$$

iii) DC diode voltage ($V_{dc, \text{diode}}$)

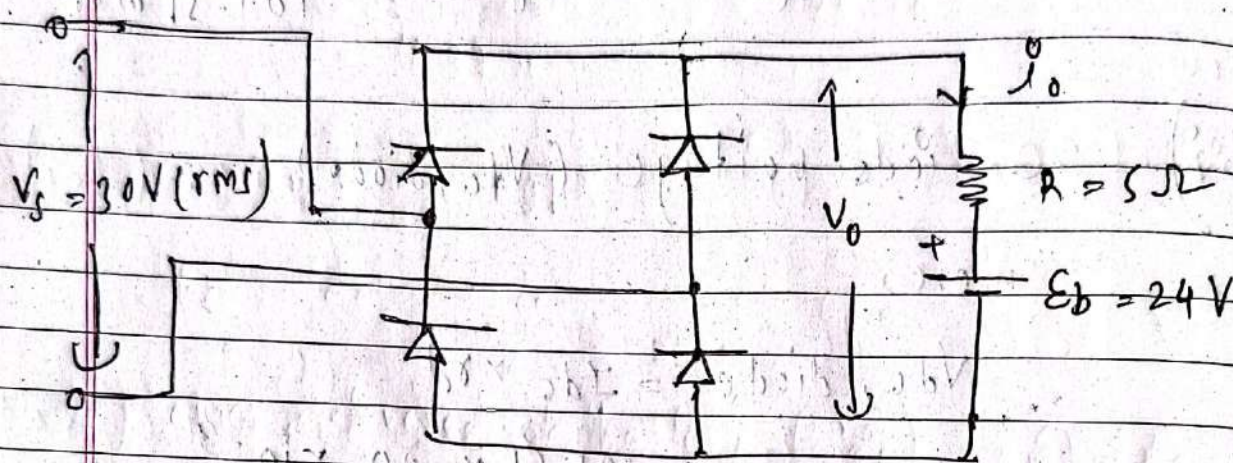
here,

$$V_{dc, \text{diode}} = I_{dc} \times r_d \\ = 101.51 \times 20 \times 10^{-3} \\ = 2.0302 \text{ V}$$

2081 Baisalkh

Date _____
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For the ckt shown, calculate 1) Avg. value of charging current 2) Efficiency of the charging system.



given,

$$V_s = 30V$$

$$V_m = \sqrt{2} \times 30 = 42.43V$$

$$E_b = 24V$$

$$R = 5\Omega$$

We have

Conduction starts when,

$$V_m \sin \alpha = E_b$$

$$\text{or, } \alpha = \sin^{-1} \left(\frac{E_b}{V_m} \right)$$

$$= \sin^{-1} \left(\frac{24}{42.43} \right)$$

$$= 0.6012 \text{ rad}$$

then, Condⁿ stops when supply falls below E_b

$$\beta = \pi - \alpha$$

$$= \pi - 0.6012$$

$$= 2.5404 \text{ rad}$$

then,

$$I_{\text{avg}} (I_{dc}) = \frac{1}{\pi} \int_{\alpha}^{\beta} \left(\frac{V_m \sin \theta - E_b}{R} \right) d\theta$$

$$= \frac{1}{\pi R} \left[V_m [\cos \alpha - \cos \beta] - E_b (\beta - \alpha) \right]$$

$$= \frac{1}{\pi \times 5} \left[42.43 [\cos 0.6012 - \cos 2.5404] - 24 [2.5404 - 0.6012] \right]$$

$$= 1.49 \text{ A}$$

then,

$$I_{rms}^2 = \frac{1}{\pi} \int_0^{\pi} \left(\frac{V_m \sin \theta - E_b}{R} \right)^2 d\theta$$

$$= 4.37$$

$$I_{rms} = 2.091 A$$

then,

$$\text{Power delivered to battery } (P_{dc}) = E_b I_{dc}$$

$$= 24 \times 1.49$$

$$= 35.76 W$$

then,

$$P_{ac} = P_{dc} + I_{rms}^2 R$$

$$= 35.76 + 2.091^2 \times 5$$

$$= 57.621 W$$

then,

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\%$$

$$= \frac{35.76}{57.621} \times 100 \%$$

$$= 62.061 \%$$

2081 Bhadra

A single phase full bridge diode rectifier is supplied from 230V, 50Hz source. The load consists of $R=10\Omega$ and a large inductance so as to render the load current constant. Determine,

- i) Avg. values of o/p voltage and o/p current
- ii) Avg and RMS values of diode currents
- iii) RMS values of o/p and i/p currents
- iv) supply pf

given,

$$230V, 50Hz$$

$$R = 10\Omega$$

then,

$$V_m = \sqrt{2} \times 230 = 325.269V$$

Load inductance (L): large enough to maintain a continuous, ripple free constant load,

$$I_o(t) = I_{dc}$$

i) Avg. values of o/p voltage and o/p current

For a single phase full bridge diode rectifier

$$V_{dc} = \frac{2V_m}{\pi} = \frac{2 \times 325.269}{\pi} \\ = 207.073V$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{207.073}{10} = 20.71A$$

ii) Avg and RMS values of diode current,

$$I_{D, avg} = \frac{I_{dc}}{2} = \frac{20.71}{2} = 10.36A$$

RMS diode current,

$$I_{D, rms} = \frac{I_{dc}}{\sqrt{2}} = \frac{20.71}{\sqrt{2}} \\ = 14.46A$$

iii) RMS values of o/p and i/p currents,

$$\rightarrow I_o(\text{rms}) = I_{dc} = 20.71 \text{ A}$$

(Because, the load current is constant and ripple free due to large inductance)

$$\rightarrow I_s(\text{rms}) = I_{dc} = 20.71 \text{ A}$$

(input)

2) supply power factor (pf)

here,

$$\begin{aligned} \text{Active power (P)} &= V \cdot I_{dc}^2 R \\ &= 20.71^2 \times 10 \\ &= 4289.041 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Apparent power (S)} &= V_i \times I_{is} \text{ rms} \\ &= 230 \times 20.71 \\ &= 4763.6 \text{ W} \end{aligned}$$

then,

$$\text{pf} = \frac{P}{S} = \frac{4289.041}{4763.6}$$

$$= 0.9003 \text{ (lagging)}$$

2079 Baisakh, 2074
Chaitra

Q. Single phase full wave rectifier charges a battery from a single phase supply of 230V, 50Hz. The battery has internal emf of 200 volts and internal resistance of 0.5Ω .

Calculate

- i) Avg. value of charging current
- ii) Power supplied to the battery
- iii) Gross power o/p from the rectifier
- iv) Additional resistance to be connected in series to reduce the charging current by 30%.

Solⁿ, given,

$$V_s = 230 \text{ V, } 50 \text{ Hz}$$

$$E_b = 200 \text{ V}$$

$$R = 0.5 \Omega$$

Then,

the battery is charged when,

$$V_m \sin \alpha = E_b$$

$$\text{or, } \alpha = \sin^{-1} \left(\frac{E_b}{V_m} \right)$$

$$= \sin^{-1} \left(\frac{200}{230\sqrt{2}} \right)$$

$$= 0.662 \text{ rad}$$

then,

$$\beta = \pi - \alpha$$

$$= \pi - 0.662$$

$$= 2.479 \text{ rad}$$

then,

$$I_{dc} = \frac{1}{\pi} \int_{\alpha}^{\beta} \left(\frac{V_m \sin \theta - E_b}{R} \right) d\theta$$

α

2.479

$$= \frac{1}{\pi} \int_{0.662}^{2.479} \left(\frac{280\sqrt{2} \sin \theta - 200}{0.5} \right) d\theta$$

0.662

$$= 95.24 \text{ A}$$

then,

$$P_{\text{battery}} = I_{dc} E_b$$

$$= 95.24 \times 200$$

$$= 19048 \text{ Watt}$$

$$= 19.05 \text{ kW}$$

Again,

$$I_{o, rms} = \frac{1}{\pi} \int_{0.662}^{2.479} \left(\frac{230\sqrt{2} \sin \theta - 200}{0.5} \right)^2 d\theta$$

$$= 18974.992$$

$$\therefore I_{o, rms} = 137.75 \text{ A}$$

then,

$$P_{gross} = P_{dc} + I_{o, rms}^2 \times R$$

(Power o/p from
the rectifier)

$$= 19048 + 137.75^2 \times 0.5$$

or

$$= 28535.53$$

P_{ac}

$$= 28.54 \text{ kW}$$

$\Rightarrow$ To reduce I_{dc} by 30%, the new current becomes,

$$I_{dc, new} = 0.7 \times I_{dc}$$

$$= 0.7 \times 95.24$$

$$= 66.668 \text{ A}$$

Since, I_{dc} is inversely proportional to total ext. resistance (R_{total}):

$$R_{total}(new) = \frac{R}{0.7} = \frac{0.5}{0.7} = 0.7143 \Omega$$

Then,

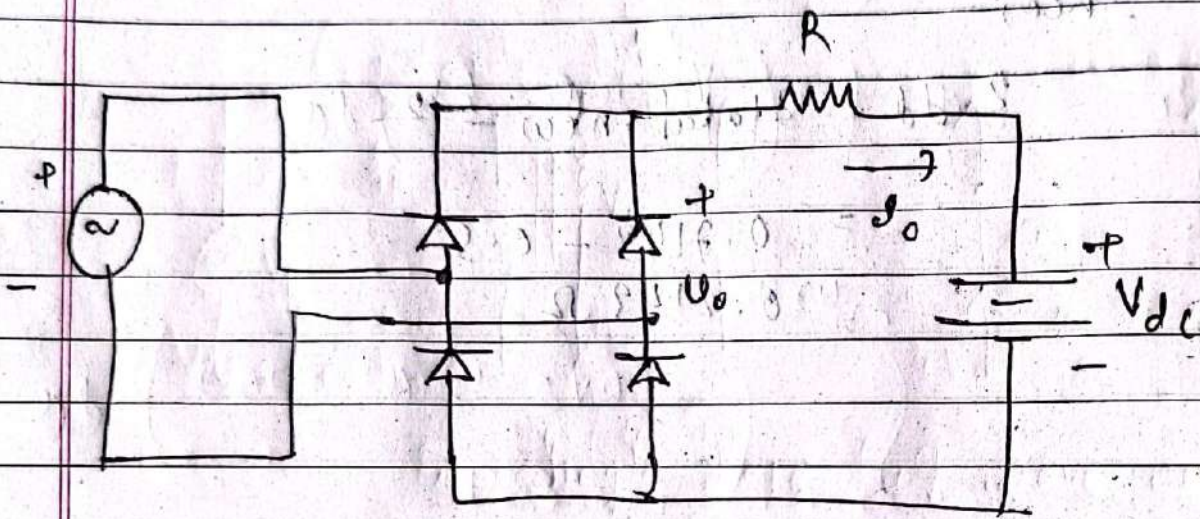
$$R_{add} = R_{total, new} - R$$

$$= 0.7143 - 0.5$$

$$= 0.2143 \Omega$$

2025 Chaitra

For the full wave bridge rectifier circuit, the AC source is 120V RMS at 50Hz , $R = 2\Omega$ and $V_{dc} = 80\text{V}$. Determine the power absorbed by the DC voltage source and the power absorbed by the load resistor R .



given,

$$V_{RMS} = 120\text{V}, 50\text{Hz}$$

$$R = 2\Omega$$

$$V_{dc} = 80\text{V}$$

then,

$$V_m \sin \alpha = V_{dc}$$

$$\text{or, } \alpha = \sin^{-1} \left(\frac{V_{dc}}{V_m} \right) = \sin^{-1} \left(\frac{80}{\sqrt{2} \times 120} \right)$$

$$= 0.49088$$

$$\beta = \pi - \alpha$$

$$= \pi - 0.49088$$

$$= 2.651 \text{ rad}$$

Then,

$$I_{dc} = \frac{1}{\pi} \int_{0.49088}^{2.651} \left(\frac{120\sqrt{2} \sin \theta - 80}{2} \right) d\theta$$

$$= 20.140 \text{ A}$$

$$I_{rms}^2 = 716.56$$

$$\therefore I_{rms} = 26.769 \text{ A}$$

Then,

$$\begin{aligned} i) P_{dc} &= 80 \times 20.140 \\ &= 1611.2 \text{ Watt} \end{aligned}$$

$$ii) P_p = I_{rms}^2 \times R$$

$$= 26.769^2 \times 2 = 1433.159 \text{ Watt}$$

2075 Chaitra

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Draw the ckt diagram and waveform of o/p voltage of a single phase full bridge diode rectifier. The input AC voltage is 220V , 50Hz . Calculate the avg value and fundamental component of o/p voltage.

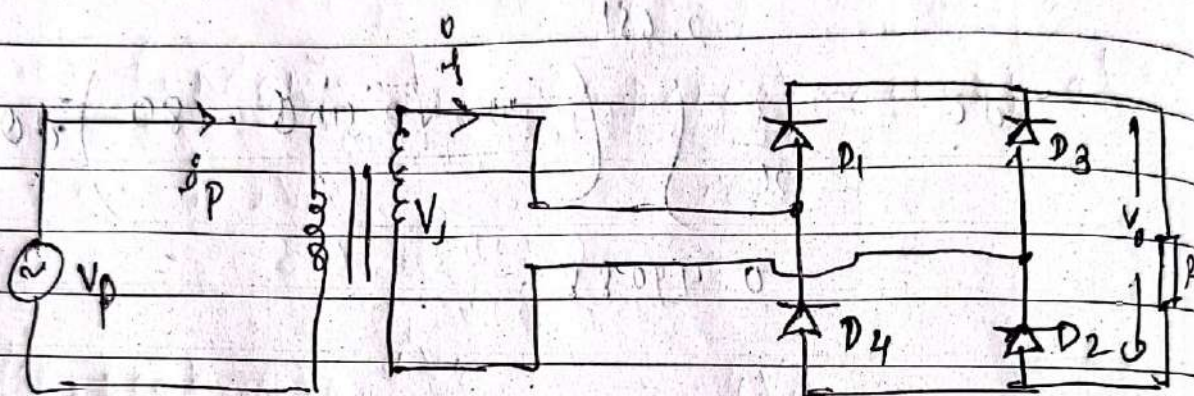
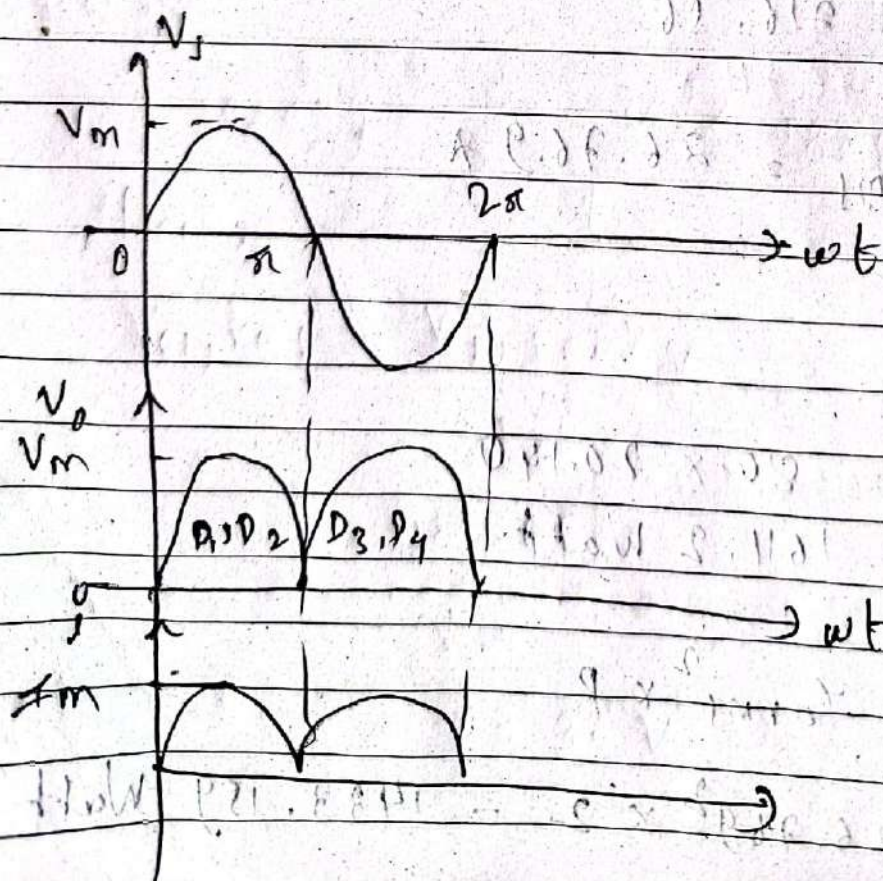


Fig. 1. Single phase full wave bridge rectifier



then,

$$V_{av} = \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t \, d\omega t$$

$$= \frac{V_m}{\pi} \left[\cos \omega t \right]_0^{\pi}$$

$$= \frac{V_m}{\pi} [\cos \pi - \cos 0]$$

$$= \frac{V_m}{\pi} [-1 - 1]$$

$$= \frac{2V_m}{\pi}$$

then,

$$1) \quad V_{avg} = \frac{2V_m}{\pi} = \frac{2\sqrt{2} \times 220}{\pi}$$

$$= 198.069 \, V$$

- 2) The o/p voltage waveform $v_o(t)$ is periodic with period π and a fundamental frequency equal to twice the supply frequency ($2f = 100 \, \text{Hz}$).

Expressed in a Fourier series,

$$V_o(t) = V_{avg} - \sum_{n=2,4,6}^{\infty} \frac{4V_m}{\pi(n^2-1)} \cos(n\omega t)$$

Here, fundamental component occurs at $n=2$. So,

$$V_{o1, peak} = \frac{4V_m}{\pi(4-1)} = \frac{4V_m}{3\pi}$$

$$= \frac{4 \times \sqrt{2} \times 220}{3\pi}$$

$$= 132.05 \text{ V}$$

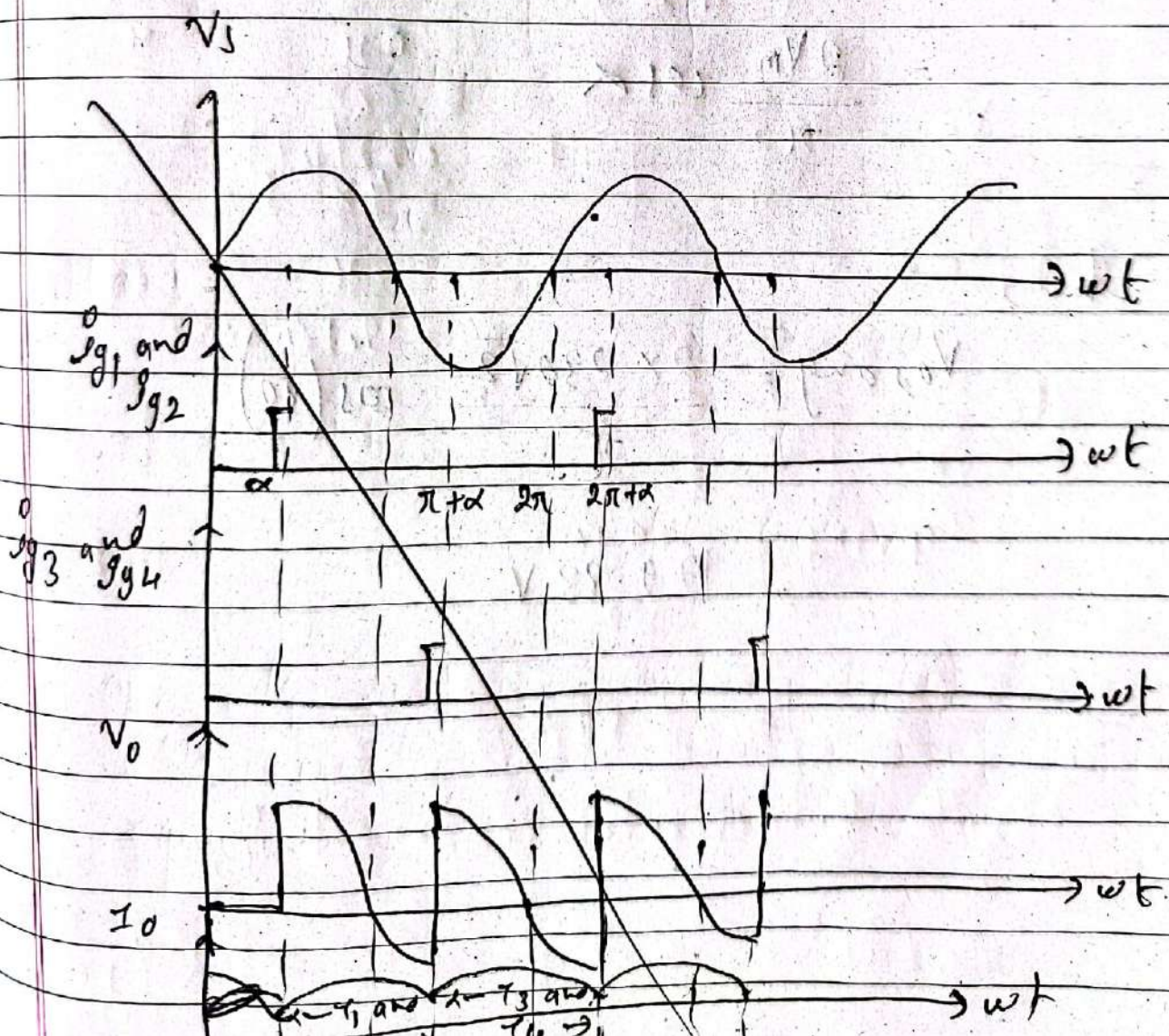
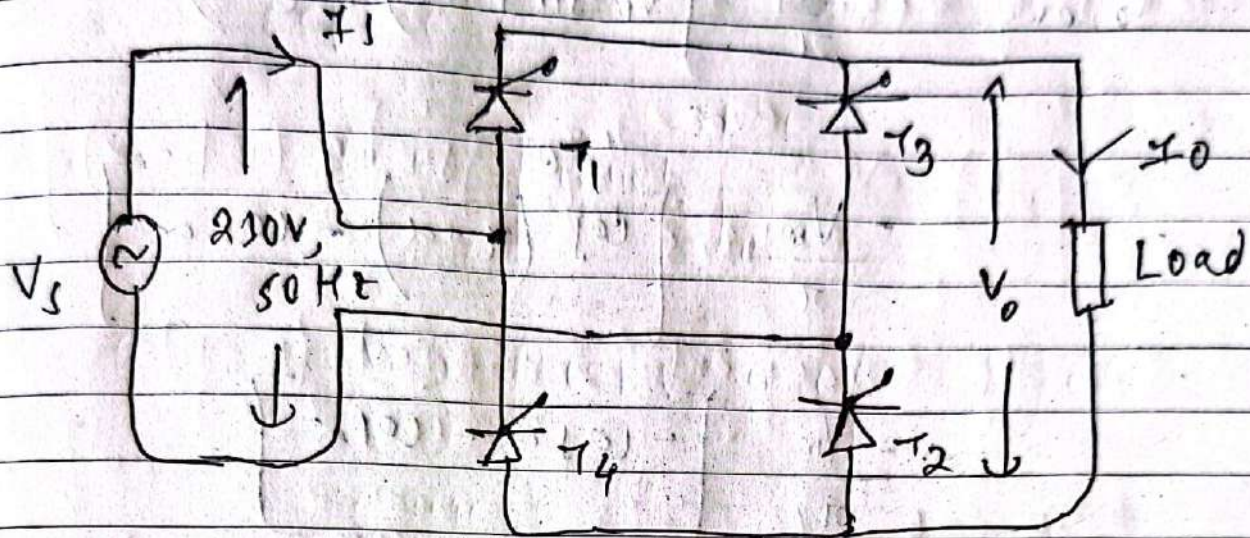
Then,

$$V_{o1, rms} = \frac{V_{o1, peak}}{\sqrt{2}}$$

$$= \frac{132.05}{\sqrt{2}}$$

$$= 93.97 \text{ V}$$

2082 Baurach



Here,

$\pi + \alpha$

$$V_{o, \text{avg}} = \frac{1}{\pi} \int_{\pi + \alpha}^{\pi} V_m \sin(\omega t) d(\omega t)$$

α

$$= -\frac{V_m}{\pi} [\cos(\pi + \alpha) - \cos \alpha]$$

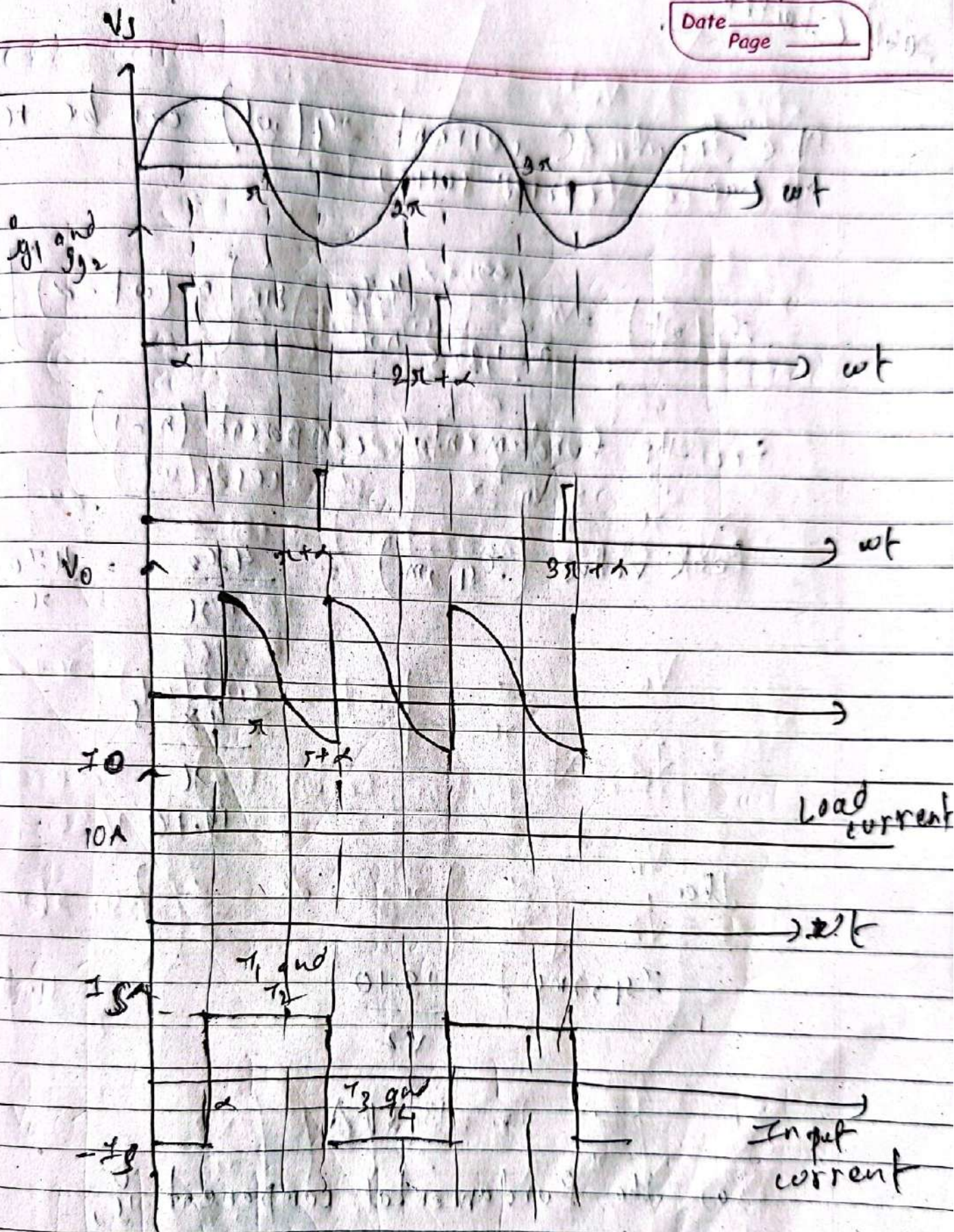
$$= -\frac{V_m}{\pi} [-\cos \alpha - \cos \alpha]$$

$$= \frac{2V_m}{\pi} \cos \alpha$$

Then,

$$V_{o, \text{avg}} = \frac{2 \times 230\sqrt{2}}{\pi} \cos(90^\circ)$$

$$= 179.33 \text{ V}$$



2024 Chaitra

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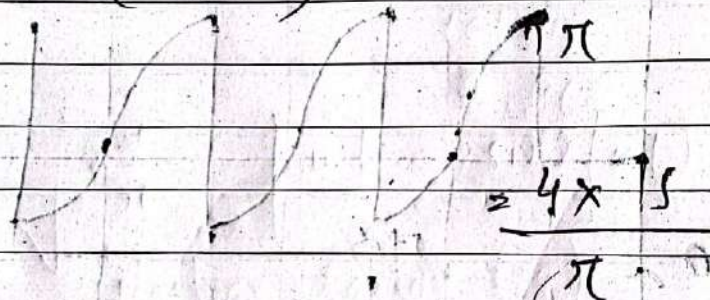
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The input AC current $I_s(\omega t)$ can be represented as a Fourier series

$$I_s(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\pi} \sin[n(\omega t - \alpha)]$$

For the fundamental component ($n=1$)

Peak value ($I_{s1,m}$) $= \frac{4I_0}{\pi} = \frac{4 \times 15}{\pi}$



$$= 19.10 \text{ V}$$

Then,

$$I_{s1,rms} = \frac{19.10}{\sqrt{2}}$$

$$= 13.51 \text{ A}$$

So, the fundamental component is,

$$I_{s1}(\omega t) = 19.10 \sin(\omega t - \alpha) \text{ A (peak form)}$$

Here,

$$V_{avg} = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t) d\omega t$$

$$= -\frac{V_m}{\pi} \left[\cos \omega t \right]_{\alpha}^{\pi+\alpha}$$

$$= -\frac{V_m}{\pi} \left[\cos(\pi+\alpha) - \cos \alpha \right]$$

$$= -\frac{V_m}{\pi} \left[-2 \cos \alpha \right]$$

$$\therefore V_{avg} = \frac{2V_m}{\pi} \cos \alpha$$

Then,

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} [V_m \sin(\omega t)]^2 d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{\pi} \int_{\alpha}^{\pi+\alpha} \left[\frac{1 - \cos 2\omega t}{2} \right] d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{4\pi}}$$

$$= \sqrt{\frac{1}{\pi} \times V_m^2 \int_{\alpha}^{\pi+\alpha} \sin^2(\omega t) d\omega t}$$

$$= \sqrt{\frac{1}{\pi} \times V_m^2 \times \int_{\alpha}^{\pi+\alpha} \frac{1 - \cos 2(\omega t)}{2} d\omega t}$$

$$= \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[\omega t - \frac{\sin 2(\omega t)}{2} \right]_{\alpha}^{\pi+\alpha}}$$

$$= \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[\pi + \cancel{\alpha} - \cancel{\alpha} - \frac{1}{2} \left[\sin 2(\pi + \alpha) - \sin 2\alpha \right] \right]}$$

$$= \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[\pi - \frac{1}{2} [0] \right]}$$

Here,

$\sin(2\pi + 2\alpha) = \sin 2\alpha$, so, the sine term cancels out.

RMS Com = $\frac{2\sqrt{2}}{n\pi} I_0$; Peak = $\frac{4I_0}{n\pi}$ 129

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$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Here

$$V = 230V, 50Hz$$

$$I = 25A$$

then,

$$V_{avg} = \frac{2V_m}{\pi} \cos \alpha$$

$$= \frac{2 \times 230\sqrt{2}}{\pi} \times \cos\left(\frac{\pi}{6}\right)$$

$$= 179.33V$$

Now, we have, For fundamental component,

$$\text{Peak value } (I_{1,m}) = \frac{4I_0}{\pi}$$

$$= \frac{4 \times 25}{\pi}$$

$$= 31.831A$$

then,

$$I_{1,rms} = \frac{31.831}{\sqrt{2}} = 22.51A$$

Now,

For Input power factor (PF)

We have,

total rms i/p current (I_s),

$$I_s = I_o = 25A$$

$$\text{Displacement factor (PF)} = \cos \alpha$$

$$= \cos 30^\circ$$

$$= 0.866$$

Now,

$$\text{Current Distortion Factor (g)} = \frac{I_1}{I_s}$$

$$= \frac{(2\sqrt{2} I_o)}{\pi}$$

$$I_o$$

$$= \frac{2\sqrt{2}}{\pi}$$

$$= 0.9003$$

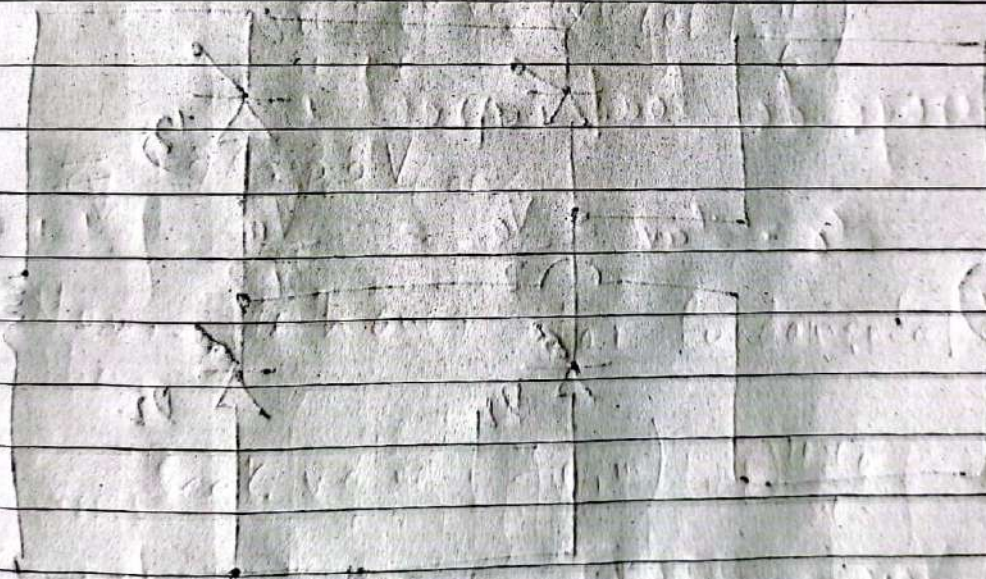
$$A 12.38$$

Then,

$$PF = g \times DF$$

$$= 0.9003 \times 0.866$$

$$= 0.7797 \text{ (lagging)}$$

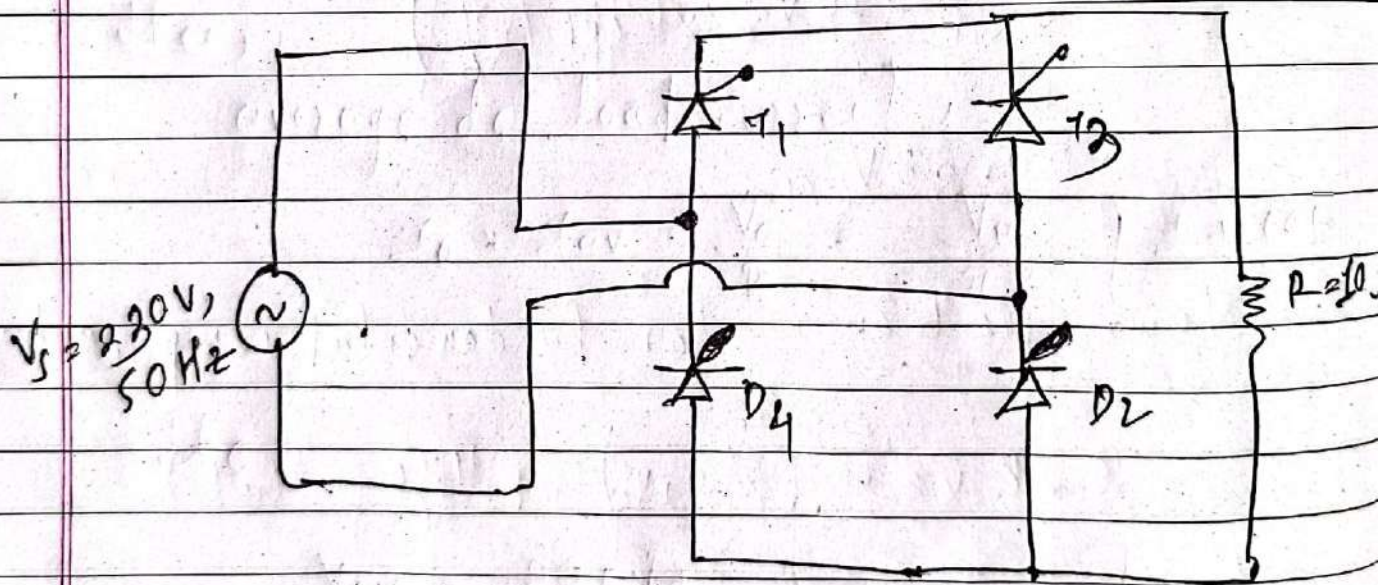


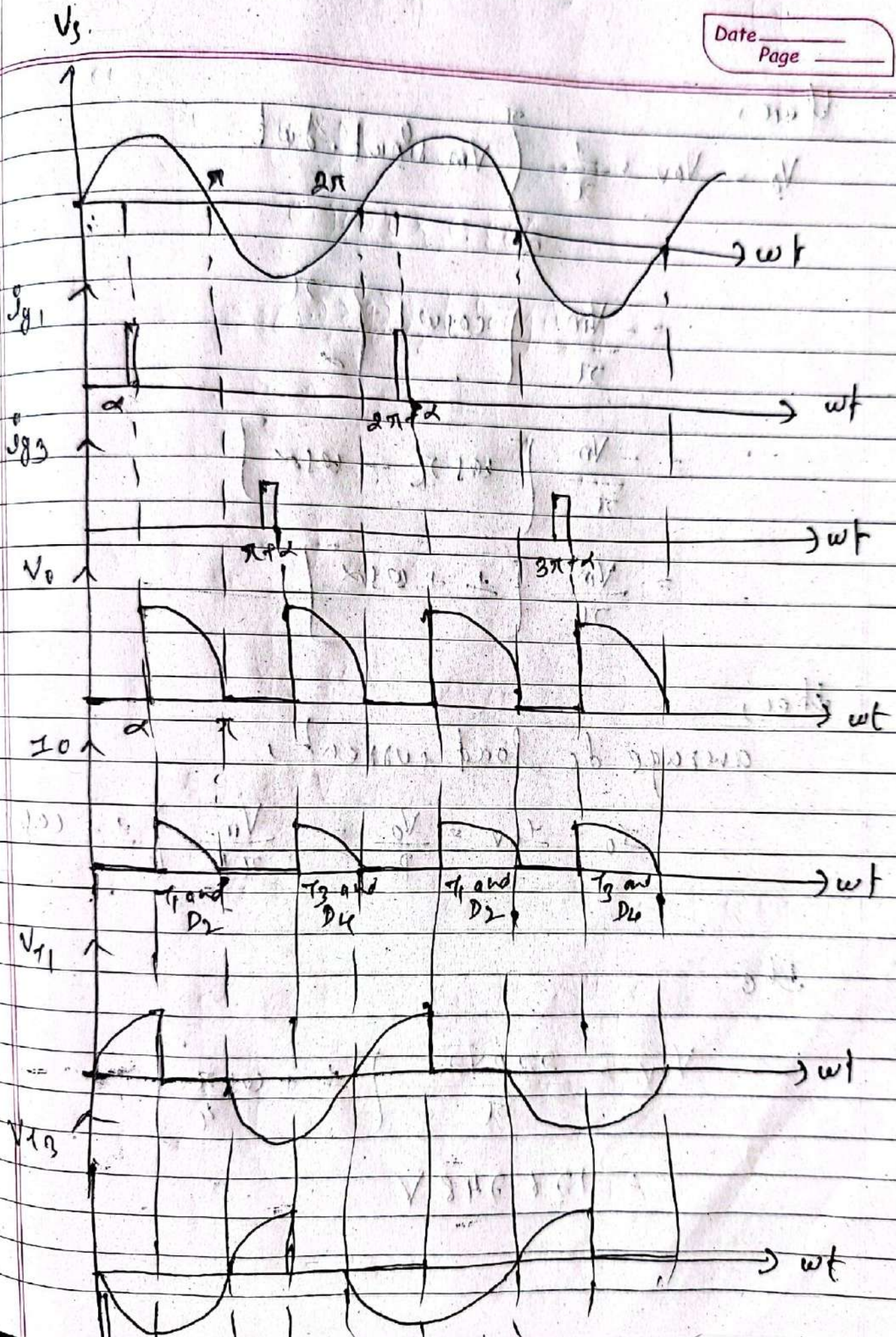
2023 Chaitra

lags exactly by the firing angle

2023 Chaitra

A full wave controlled rectifier with resistive load, load voltage 230V, 50Hz, fired at 45° . Draw the waveforms of o/p voltage, o/p current and AC i/p current. Calculate avg. value of o/p voltage, o/p DC power and i/p power factor.





Here,

$$V_0 = V_{av} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \cdot d\omega t$$

$$= -\frac{V_m}{\pi} \left[\cos \omega t \right]_{\alpha}^{\pi}$$

$$= -\frac{V_m}{\pi} \left[\cos \pi - \cos \alpha \right]$$

$$= \frac{V_m}{\pi} \left[1 + \cos \alpha \right]$$

then,

average dc load current,

$$I_0 = I_{av} = \frac{V_0}{R} = \frac{V_m}{\pi R} \left[1 + \cos \alpha \right]$$

the

$$V_{av} = \frac{230\sqrt{2}}{\pi} \left[1 + \cos \frac{\pi}{4} \right]$$

$$= 171.748 \text{ V}$$

then,

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$$I_{av} = \frac{176.748}{10}$$

$$= 17.674 \text{ A}$$

then,

$$P_{dc} = V_{dc} I_{dc}$$

$$= 176.748 \times 17.674$$

$$= 3123.844$$

$$= 3.124 \text{ kW}$$

then,

$$V_{o, rms} = \sqrt{\frac{1}{\pi} \int_0^{\pi} [230\sqrt{2} \times \sin \omega t]^2 d\omega t}$$

$$= 219.30 \text{ V}$$

$$I_{o, rms} = \frac{219.30}{10} = 21.93 \text{ A}$$

then,

$$P_o = \frac{V_{o, rms}^2}{R} = \frac{219.30^2}{10} = 4809.249 \text{ W}$$

then,

$$PF = \frac{P}{V_i I_{s, rms}} = \frac{4809.249}{230 \times 21.93} = 0.9534 \text{ (lagging)}$$