

Control of Electric Drive

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56) given,

230 V, 25 kW DC shunt motor

$$N = 1500 \text{ rpm}$$

$$R_a = 0.1 \Omega$$

$$\eta = 88\% = 0.88$$

We have,

$$\text{Rated current} = \frac{25 \times 10^3}{230 \times 0.88}$$

$$= 123.52 \text{ A}$$

We have,

for Normal motoring,

$$V = E_b + I_a R_a \quad \text{--- (1)}$$

$$\text{or, } E_b = V - I_a R_a$$

$$\text{or, } E_b = 230 - 123.52 \times 0.1$$

$$\therefore E_b = 217.648 \text{ V}$$

Let, R_b be the braking resistance to be connected in series, with the armature.

then,

$$\begin{aligned} \text{Braking armature current} &= 2 \times 123.52 \\ &= 247.04 \text{ A} \end{aligned}$$

During braking,

V is replaced by $-V$
 I_a is replaced with $-I_a$ in (i) with
 R_b in series. We get,

$$\text{or, } -V = E_b - I_a (R_a + R_b)$$

$$\text{or, } V + E_b = I_a (R_a + R_b)$$

$$\text{or, } 230 + 217.648 = 247.04 (0.1 + R_b)$$

$$\text{or, } 0.1 + R_b = 1.812$$

$$\therefore R_b = 1.712 \Omega$$

ii) For initial braking torque,

$$P_b = \tau_b \omega$$

$$\text{or, } \tau_b = \frac{P_b}{\omega}$$

$$= \frac{P_b}{\left(\frac{2\pi N}{60} \right)} = \frac{217.648 \times 247.04}{\left(\frac{2\pi \times 1500}{60} \right)}$$

$$= 342.296 \text{ Nm}$$

For a DC motor,

$$E_b \propto \phi N$$

Since ϕ is constant,

$$E_b \propto N$$

$$\text{or, } \frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

When speed reduces to 600 rpm,

$$E_{b2} = \frac{600}{1500} \times 217.648$$

$$\therefore E_{b2} = 87.06 \text{ V}$$

Then,

Armature current I_a is given by,

$$I_a = \frac{V + E_{b2}}{R_a + R_b}$$

$$= \frac{230 + 87.06}{0.1 + 1.712}$$

$$= 174.978 \text{ A}$$

ii)

Since, In shunt motor

$$T \propto \phi I_a$$

and, ϕ remains constant. So,

$$T \propto I_a$$

$$\text{or, } \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$$\text{or, } T_2 = \frac{I_{a2}}{I_{a1}} \times T_1$$

$$\text{or, } T_2 = \frac{174.978}{247.04} \times 342.296$$

$$\therefore T_2 = 242.448 \text{ Nm}$$

57) Here, given,

220V, Separately excited motor,

$$N = 1000 \text{ rpm}$$

$$I_a = 100 \text{ A}$$

Motor is, Braked by plugging.

Since, armature resistance is not given, assuming armature resistance (R_a) = 0.1Ω

Now, for normal motoring,

$$V = E_b + I_a R_a \quad \text{--- (i)}$$

$$\text{or, } E_b = V - I_a R_a$$

$$\text{or, } E_b = 220 - 100 \times 0.1$$

$$\therefore E_b = 210 \text{ V}$$

Now, after braking (plugging)

the equation becomes,

$$V + E_b = I_a' (R_a + R_b) \quad \text{--- (ii)}$$

where,

R_b is the braking resistance to be inserted in the armature circuit.

also,

$$I_a' = 2 \times I_a$$

$$= 2 \times 100$$

$$= 200 \text{ A}$$

Then, (ii) becomes,

$$220 + 210 = 200 (0.1 + R_b)$$

$$\text{or, } 0.1 + R_b = 2.15$$

$$\therefore R_b = 2.05 \Omega$$

ii) For initial braking torque,

$$T_b = \frac{P_b}{\omega}$$

$$\text{or, } T_b = \frac{E_b \times I_a}{\left(\frac{2\pi N}{60} \right)}$$

$$= \frac{210 \times 200}{\left(\frac{2\pi \times 1000}{60} \right)}$$

$$= 401.07 \text{ Nm}$$

iii) We have,

$$E_b \propto \phi N$$

since ϕ is constant,

$$E_b \propto N$$

when speed reduces to 500 rpm. then,

$$E_{b2} = \frac{N_2}{N_1} \times E_{b1}$$

$$= \frac{500}{1000} \times 210$$

$$= 105 \text{ V}$$

Then,

$$\begin{aligned} I_a'' &= \frac{V + E_{ba}}{R_a + R_b} \\ &= \frac{220 + 105}{0.1 + 2.05} \\ &= 151.163 \text{ A} \end{aligned}$$

In separately excited motor, field remains constant. So,

$$T \propto \phi I_a \propto I_a$$

i.e.

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$$\text{or, } T_2 = \frac{151.163}{200} \times 401.07$$

$$T_2 = 303.135 \text{ Nm}$$

59) Given,

200V, 1000 rpm, 50A

(separately excited dc motor)

i.e.

$$N_1 = 1000 \text{ rpm}$$

$$N_2 = 0$$

$$R_a = 0.2 \Omega$$

We have,

$$V = E_b + I_a R_a$$

$$\text{or, } E_b = V - I_a R_a$$

$$\text{or, } E_b = 200 - 50(0.2)$$

$$\therefore E_b = \cancel{210V} = 190V$$

Now,

$$\begin{aligned} \text{Braking current } (I_{a1}) &= 2 \times 50 \\ &= 100 \text{ A} \end{aligned}$$

1) After plugging, the eqn of motor becomes,

$$V + E_b = I_{a1}(R_a + R_b)$$

where,

R_b is the additional resistance to be connected in series with armature.

then,

$$\therefore 200 \pm 90$$

$$\text{or, } \cancel{200} + \cancel{210} = 100 \cdot (0.2 + R_b)$$

$$\text{or, } 0.2 + R_b = \cancel{4} + 3.9$$

$$\therefore R_b = \cancel{3.9 \Omega} = 3.7 \Omega$$

ii) For braking torque,

$$T_b = \frac{P_b}{\omega} = \frac{E_b \times I_{a1}}{\left(\frac{2\pi N}{60} \right)}$$

$$= \frac{\cancel{210} \times 100}{\left(\frac{2\pi \times 1000}{60} \right)}$$

$$\therefore T_b = \cancel{200.535 \text{ Nm}} = 181.437 \text{ Nm}$$

i.e.

Initial braking torque is 181.437 Nm

iii) When speed decreases to zero,

$$N = 0$$

i.e.

$$E_b = 0$$

So,

$$I_{a2} = \frac{V}{R_a + R_b} = \frac{200}{0.2 + 3.7}$$

$$\therefore I_{a2} = 51.282 \text{ A}$$

Since the field flux remains constant,

$$\tau \propto I_a$$

$$\text{or, } \frac{\tau_2}{\tau_1} = \frac{I_{a2}}{I_{a1}}$$

$$\text{or, } \tau_2 = \frac{I_{a2}}{I_{a1}} \times \tau_1$$

$$\text{or, } \tau_2 = \frac{51.282}{100} \times 181.437$$

$$\therefore \tau_2 = 93.044 \text{ Nm}$$

60) Given,

$$\text{Supply Voltage (V)} = 220 \text{ V}$$

$$\text{Motor efficiency } (\eta_m) = 85\% = 0.85$$

$$\text{Hoist efficiency } (\eta_h) = 80\% = 0.80$$

$$\text{Load mass (m)} = 400 \text{ kg}$$

$$\text{Speed of hoisting / lowering (v)} = 2.5 \text{ m/s}$$

Taking,

$$g = 9.8 \text{ m/s}^2$$

9) Current drawn from the supply while raising the load,

The useful power required to raise the load,

$$P_L = mgv$$

$$= 400 \times 9.81 \times 2.5$$

$$= 9810 \text{ W}$$

$$= 9.81 \text{ kW}$$

Then,

$$\text{Hoist efficiency } (\eta_h) = \frac{\text{Power delivered to the load } (P_L)}{\text{Motor shaft o/p}}$$

Motor shaft o/p

$$\text{or, } P_{\text{shaft}} = \frac{P_L}{\eta_h} = \frac{9810}{0.8}$$

$$\therefore P_{\text{shaft}} = 12262.5 \text{ W}$$

$$= 12.2625 \text{ kW}$$

again,

$$\text{Motor efficiency } (\eta_m) = \frac{\text{Mechanical o/p}}{\text{Electrical i/p}}$$

$$\text{or, } \eta_m = \frac{P_{\text{shaft}}}{P_{\text{in}}}$$

$$\text{or, } P_{\text{in}} = \frac{12262.5}{0.85}$$

$$\therefore P_{\text{in}} = 14426.47 \text{ W} \\ = 14.426 \text{ kW}$$

Now,

for the DC motor,

$$P_{\text{in}} = VI$$

$$\text{or, } I = \frac{P_{\text{in}}}{V}$$

$$\text{or, } I = \frac{14426.47}{220}$$

$$\therefore I = 65.575 \text{ A}$$

$\therefore$ Current drawn while raising the load is 65.575 A.

Now,

ii) Resistance required for lowering using rheostatic braking,

→ In rheostatic braking, the motor is disconnected from the supply and operates as a generator.

The mechanical energy of the descending load is converted into electrical energy and dissipated as heat in the braking resistance.

Now,

Power available from descending load,

$$\begin{aligned} P_L &= mgv \\ &= 400 \times 9.81 \times 2.5 \\ &= 9810 \text{ W} \\ &= 9.81 \text{ kW} \end{aligned}$$

This is the mechanical power supplied by the load to the hoist.

During lowering, power transmission takes place in the reverse direction through the hoist.

So,

$$\eta_h = \frac{P_{\text{shaft (O/P)}}}{P_L \text{ (I/P)}}$$

$$\text{or, } P_{\text{shaft}} = 0.8 \times 9810 = 7848 \text{ W} = 7.848 \text{ kW}$$

Similarly,
During rheostatic braking, motor acts as generator.

So,

$$\eta_m = \frac{P_g \text{ (o/p)}}{P_{\text{shaft}} \text{ (i/p)}}$$

$$\text{or, } P_g = 0.85 \times 7848$$

$$\therefore P_g = 6670.8 \text{ W}$$

$$= 6.6708 \text{ kW}$$

For DC shunt machine with constant excitation,

$$E \propto N$$

Since, speed during lowering is the same as that during raising, assuming constant flux,

$$E = 220 \text{ V.}$$

So,

the generated emf is

$$E = 220 \text{ V}$$

Now,

Generated electrical power,

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$$P_g = EI$$

$$\text{or, } I = \frac{P_g}{E} = \frac{6670.8}{220}$$

$$\therefore I = 30.322 \text{ A}$$

For rheostatic braking,

$$I = \frac{E}{R_{\text{total}}}$$

$$\text{or, } R_{\text{total}} = \frac{E}{I} = \frac{220}{30.322}$$

$$\therefore R_{\text{total}} = 7.26 \Omega$$

$\therefore$ Resistance required for rheostatic braking = 7.26Ω

61) Given,

$$J = 10 \text{ kg m}^2$$

Motor torque (Forward), $\tau = 100 - 0.1 \text{ N Nm}$

Passive load torque, $\tau_L = 0.05 \text{ N Nm}$

Motor torque (after reversal),

$$\tau = -100 - 0.1 \text{ N Nm}$$

Now,

In steady state, $\frac{dN}{dt} = 0$. So,

$$\tau = \tau_L$$

$$\text{or, } 100 - 0.1N =$$

Initial Steady State Speed (N_1)

In steady state, $\frac{dN}{dt} = 0$. So,

$$\tau = \tau_L$$

$$\text{or, } 100 - 0.1N_1 = 0.05N_1$$

$$\text{or, } 0.15N_1 = 100$$

$$\text{or, } N_1 = 666.67 \text{ rpm}$$

Final Steady state Speed after reversal (N_2)

After reversal, new steady state is reached when $\tau = \tau_L$ again (with the new characteristic).

$$-100 - 0.1N_2 = 0.05N_2$$

$$\text{or, } -100 = 0.15 N_2$$

$$\therefore N_2 = -666.67 \text{ rpm}$$

Since, the system reaches exact steady state only as $t \rightarrow \infty$, transient (reversal) time is conventionally taken as the time to reach 95% of the final steady-state speed.

$$N_f = 0.95 \times N_2$$

$$= 0.95 \times (-666.67)$$

$$= -633.33 \text{ rpm}$$

Now,

Fundamental torque equation is given by,

$$\tau - \tau_L = J \frac{d\omega_m}{dt} = J \frac{d}{dt} \left(\frac{2\pi N}{60} \right)$$

$$= J \left(\frac{2\pi}{60} \right) \frac{dN}{dt}$$

During reversal, motor torque is,

$\tau = -100 - 0.1 \text{ N}$, substituting. We get,

$$\frac{2\pi J}{60} \frac{dN}{dt} = (-100 - 0.1N) - 0.05N$$
$$= -(100 + 0.15N)$$

Then

$$dt = \frac{2\pi J}{60} \frac{dN}{-(100 + 0.15N)}$$

Integrating from N_i to N_f . We get,

$$t = \frac{2\pi J}{60} \int_{N_i}^{N_f} \frac{-dN}{100 + 0.15N}$$

$$= - \frac{2\pi \times 10}{60 \times 0.15} \left[\ln(100 + 0.15N) \right]_{N_i}^{N_f}$$

$$= - \frac{2\pi \times 10}{60 \times 0.15} \left[\ln(100 + 0.15N_f) - \ln(100 + 0.15N_i) \right]$$

$$= \frac{2\pi \times 10}{60 \times 0.15} \left[\ln(100 + 0.15N_i) - \ln(100 + 0.15N_f) \right]$$

$$= \frac{2\pi \times 10}{60 \times 0.15} \ln \left[\frac{100 + 0.15 \times 666.67}{100 + 0.15 \times (-693.33)} \right]$$

$$= 6.9813 \times \ln [39.996]$$

$$\therefore t = 25.752 \text{ sec}$$

$\therefore$ Time of reversal is 25.752 sec.

12) Given,

30 kW, 220V, DC shunt motor

Full load speed (N) = 535 rpm

(To be braked by plugging)

$$R_a = 0.086 \Omega$$

$$I_a = 150 \text{ A}$$

for normal motoring,

$$V = E_b + I_a R_a$$

$$\text{or, } E_b = V - I_a R_a$$

$$= 220 - 150 \times 0.086$$

$$= 207.1 \text{ V}$$

We have,

$$\text{Braking current } (I_a') = 200 \text{ A}$$

then, after plugging,

$$I_a' (R_a + R_b) = V + E_b$$

$$\therefore, 0.086 + R_b = \frac{220 + 207.1}{200}$$

$$\text{or, } 0.086 + R_b = 2.1355$$

$$\therefore R_b = 2.0495 \Omega$$

then,

Initial value of braking torque,

$$T_b = \frac{P_b}{\omega}$$

$$= \frac{E_b \times I_a'}{\left(\frac{2\pi N}{60} \right)}$$

$$= \frac{207.1 \times 200}{\left(\frac{2\pi \times 535}{60} \right)}$$

$$= 739.31 \text{ Nm}$$

Now, when speed reduces to half of full load value,

$$\text{i.e. } N_2 = \frac{535}{2} = 267.5 \text{ rpm}$$

We have,

$$E_b \propto N$$

$$\text{or, } \frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\text{or, } E_{b2} = \frac{N_2}{N_1} \times E_{b1}$$

$$= \frac{267.5}{535} \times 207.1$$

$$\therefore E_{b2} = 103.55 \text{ V}$$

Then,

Armature current,

$$I_a'' = \frac{V + E_{b2}}{R_a + R_b}$$

$$= \frac{220 + 103.55}{0.086 + 2.0495}$$

$$= 151.51 \text{ A}$$

We have,

$$\tau \propto I_a$$

Since ϕ is constant in shunt motor. So,

$$\frac{\tau_2}{\tau_1} = \frac{I_{a2}}{I_{a1}}$$

$$\text{or, } \tau_2 = \frac{I_{a2}}{I_{a1}} \times \tau_1$$

$$\text{or, } \tau_2 = \frac{151.51}{200} \times 739.31$$

$$\therefore \tau_2 = 560.064 \text{ Nm}$$

64) given,

550V, 45kW, 600 rpm,

(dc shunt motor)

full load efficiency = 90% = 0.90

field resistance (R_f) = 200 Ω

armature resistance (R_a) = 0.2 Ω

then,

Rated line current,

$$I_L = \frac{\text{Output } P_n \text{ kW} \times 1000}{V \times \eta}$$

$$= \frac{45 \times 10^3}{550 \times 0.90}$$

$$= 90.91 \text{ A}$$

Shunt field current,

$$I_f = \frac{V}{R_f} = \frac{550}{200} = 2.75 \text{ A}$$

Armature current on full load,

$$\begin{aligned} I_a &= I_L - I_f \\ &= 90.91 - 2.75 \\ &= 88.16 \text{ A} \end{aligned}$$

Induced emf on full load,

$$\begin{aligned} E &= V - I_a R_a \\ &= 550 - 88.16 \times 0.2 \\ &= 532.368 \text{ V} \end{aligned}$$

Rated speed (N) = 600 rpm

As torque to be developed in each case is equal to the rated torque, so, armature current in each case is equal to armature current at full load.

$$\text{i.e. } I_a = \cancel{99} 88.16 \text{ A}$$

i) Regenerative braking with no limiting resistance:

During regenerative braking, motor acts as a generator. So,

$$E_g = V + I_a R_a$$

$$= 550 + 88.16 \times 0.2$$

$$= 567.632 \text{ V}$$

Then,

$$N \propto E$$

$$\text{or, } \frac{N_g}{N} = \frac{E_g}{E}$$

$$\text{or, } N_g = \frac{E_g}{E} \times N$$

$$= \frac{567.632}{532.368} \times 600$$

$$\therefore N_g = 639.744 \text{ rpm}$$

ii) Plugging with external limiting resistance of 5.5Ω inserted:

$$\begin{aligned} E_p &= -V + I_a (R_a + 5.5) \\ &= -550 + 88.16 (0.2 + 5.5) \\ &= -47.488 \text{ V} \end{aligned}$$

then,

$$\frac{N_p}{N} = \frac{E_p}{E}$$

$$\text{or, } N_p = \frac{E_p}{E} \times N$$

$$\text{or, } N_p = \frac{-47.488}{532.368} \times 600$$

$$\therefore N_p = -53.521 \text{ rpm}$$

i.e. 53.521 rpm in the reverse direction

iii) Dynamic braking with external limiting resistance of 2.6Ω inserted.

→ In dynamic braking, the motor is disconnected from the supply and operates as a generator, with the generated power dissipated in the external resistance.

We have,

$$E_d = I_a (R_a + 2.6)$$

$$= 88.16 (0.2 + 2.6)$$

$$= 246.842 \text{ V}$$

Then,

$$N \propto E$$

$$\text{or, } \frac{N_d}{N} = \frac{E_d}{E}$$

$$\text{or, } N_d = \frac{E_d}{E} \times N$$

$$\text{or, } N_d = \frac{246.842}{532.368} \times 600$$

$$\therefore N_d = 278.2076 \text{ rpm}$$

65) Given that,

$$\tau_m = \tau (1 + 2\omega_m)$$

$$\tau_L = -3\sqrt{\omega_m}$$

For an electric drive, the fundamental dynamic equation of motion is,

$$\tau_m - \tau_L = J \frac{d\omega_m}{dt}$$

Equilibrium point occurs when $\frac{d\omega_m}{dt} = 0$. i.e.

$$\tau_m = \tau_L$$

$$\text{or, } -(1 + 2\omega_m) = -3\sqrt{\omega_m}$$

$$\text{or, } 1 + 2\omega_m = 3\sqrt{\omega_m}$$

Let,

$$x = \sqrt{\omega_m} \text{ . Then,}$$

$$\text{or, } 1 + 2x^2 = 3x$$

$$\text{or, } 2x^2 - 3x + 1 = 0$$

Solving. We get,

$$x = 1 \quad \text{or, } x = \frac{1}{2}$$

$$\text{Since, } x = \sqrt{\omega_m} \text{ . So,}$$

$$1 = \sqrt{\omega_m}$$

$$\therefore \omega_m = 1 \text{ rad/s}$$

$$\left\{ \begin{array}{l} \frac{1}{2} = \sqrt{\omega_m} \\ \text{or, } \omega_m = \frac{1}{4} \end{array} \right.$$

$$\therefore \omega_m = 0.25 \text{ rad/s}$$

i.e.

there are two equilibrium points. i.e.

$$\omega_{m1} = 1 \text{ rad/s}$$

$$\omega_{m2} = 0.25 \text{ rad/s}$$

Now,

For stability criterion,

$$\Rightarrow \frac{dT_m}{d\omega_m} = \frac{d}{d\omega_m} [-1 - 2\omega_m]$$

$$\therefore \frac{dT_m}{d\omega_m} = -2$$

$$\begin{aligned} \Rightarrow \frac{dT_L}{d\omega_m} &= \frac{d}{d\omega_m} [-3\sqrt{\omega_m}] \\ &= -3 \times \frac{1}{2\sqrt{\omega_m}} \end{aligned}$$

$$\therefore \frac{dT_L}{d\omega_m} = \frac{-3}{2\sqrt{\omega_m}}$$

Now,

The stability criterion is,

$$\left[\frac{dT_m}{d\omega_m} - \frac{dT_L}{d\omega_m} \right] < 0 \Rightarrow \text{stable}$$

$$\text{or, } -2 - \left[\frac{-3}{2\sqrt{\omega_m}} \right] = -2 + \frac{3}{2\sqrt{\omega_m}}$$

$$\Rightarrow \text{At } \omega_{m1} = 1 \text{ rad/s}$$

$$\frac{-2 + 3}{2\sqrt{\omega_m}} = \frac{-2 + 3}{2} = -0.5 < 0 \quad (\text{stable})$$

Since the value is less than 0, the equilibrium point, $\omega_{m1} = 1 \text{ rad/s}$ is stable.

$$\Rightarrow \text{At } \omega_{m2} = 0.25 \text{ rad/s}$$

$$\frac{-2 + 3}{2\sqrt{0.25}} = 1 > 0 \quad (\text{Unstable})$$

Since, the value is greater than 0, the equilibrium point, $\omega_{m2} = 0.25 \text{ rad/s}$ is unstable.

66) Theory (For Understanding)

For an electric drive, the equation of motion is,

$$J \frac{d\omega}{dt} = T_m - T_L$$

"At eqb",

$$\frac{d\omega}{dt} = 0$$

therefore,

$$T_m = T_L$$

The four quadrants of operation are

Quadrant	Speed	Torque	Mode
I	$N > 0$	$T > 0$	Forward motoring
II	$N > 0$	$T < 0$	Forward braking
III	$N < 0$	$T < 0$	Reverse motoring
IV	$N < 0$	$T > 0$	Reverse braking

Solution, given,

Motor torque characteristics,

$$T_{m1} = 200 - 0.2N \text{ Nm (I, II and IV)}$$

$$T_{m2} = -200 - 0.2N \text{ Nm (II, III and IV)}$$

Load torques,

$$\tau_{L1} = 100 \text{ Nm} \quad (\text{hoist is loaded})$$

$$\tau_{L2} = -80 \text{ Nm} \quad (\text{hoist is unloaded})$$

Now,

⇒ Quadrant I: Forward motoring ($N > 0, \tau > 0$)
Condition: Raising a loaded hoist.

We have,

$$\tau_{m1} = \tau_{L1}$$

$$\text{or, } 200 - 0.2N = 100$$

$$\text{or, } 100 = 0.2N$$

$$\therefore N = +500 \text{ rpm}$$

Here, $N > 0, \tau > 0$ (satisfies Quadrant I)

⇒ Quadrant II: Forward braking ($N > 0, \tau < 0$)

We have,

$$\tau_{m1} = \tau_{L2}$$

$$\text{or, } 200 - 0.2N = -80$$

$$\text{or, } 280 = 0.2N$$

$$\therefore N = +1400 \text{ rpm}$$

Here, $N > 0$, $T < 0$ (Controlled raising of an unloaded hoist)

⇒ Quadrant III : ($N < 0$, $T < 0$), Reverse motor
Here, at eqb^m,

$$T_{m2} = T_{L2}$$

$$\text{or, } -200 - 0.2N = -80$$

$$\text{or, } 0.2N = -120$$

$$\therefore N = -600 \text{ rpm}$$

Here, $N < 0$, $T < 0$ (Lowering of an unloaded hoist)

⇒ Quadrant IV : ($N < 0$, $T > 0$), Reverse braking
Here, at eqb^m,

$$T_{m2} = T_{L1}$$

$$\text{or, } -200 - 0.2N = 100$$

$$\text{or, } -200 - 100 = +0.2N$$

$$\therefore N = -1500 \text{ rpm}$$

Here, $N < 0$, $T > 0$ (Controlled lowering of a loaded hoist)

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For Conclusion in quadrants remember,

$$T > 0 \text{ (loaded)}$$

$$T < 0 \text{ (unloaded)}$$

$$N > 0 \text{ (raising)}$$

$$N < 0 \text{ (lowering)}$$

Braking mode (controlled)

76) Here, given parameters,

$$\text{Armature current } (I_a) = 60 \text{ A}$$

$$\text{Armature resistance } (R_a) = 0.5 \Omega$$

$$\text{Rated speed } (N_1) = 1200 \text{ rpm}$$

$$\text{Rated terminal voltage } (V_b) = 400 \text{ V}$$

Now, Back EMF at rated speed is given by,

$$E_{b1} = V - I_a R_a$$

$$\text{or, } E_{b1} = 400 - 60 \times 0.5$$

$$\therefore E_{b1} = 370 \text{ V}$$

Now,

Determining RMS line to line supply voltage (V_L)
for $\alpha = 0^\circ$

For a three phase six pulse converter, the average output DC voltage (V_o) is given by,

$$V_o = \frac{3\sqrt{2} V_L}{\pi} \cos \alpha$$

At $\alpha = 0^\circ$, the converter o/p voltage equals the motor rated terminal voltage, $V = 400V$.

i.e.

$$400 = \frac{3\sqrt{2} V_L}{\pi} \cos 0$$

$$\text{or, } 400 = \frac{3\sqrt{2} V_L}{\pi}$$

$$\text{or, } V_L = \frac{400\pi}{3\sqrt{2}}$$

$$\therefore V_L = 296.192 V$$

Now,

⇒ For the range of firing angles (α) for speed control

from 1800 rpm to 900 rpm

At $N_1 = 1800$ rpm

$$V_a = 400 \text{ V}$$

At $N_2 = 900$ rpm

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\text{or, } E_{b2} = \frac{N_2}{N_1} \times E_{b1}$$

$$\text{or, } E_{b2} = \frac{900}{1800} \times 370$$

$$\therefore E_{b2} = 185 \text{ V}$$

Now,

Assuming the motor continues to deliver full-load (rated) torque, armature current remains the same,

$$I_a = 60$$

Then,

$$\begin{aligned} V_{a2} &= E_{b2} + I_a R_a \\ &= 185 + 60 \times 0.5 \\ &= 215 \text{ V} \end{aligned}$$

Now for α_2 (at 900 rpm),

$$V_{a2} = \frac{3\sqrt{2} V_L}{\pi} \cos \alpha_2$$

$$\text{or, } 215 = \frac{3\sqrt{2} \times 296.192}{\pi} \times \cos \alpha_2$$

$$\text{or, } \cos \alpha_2 = \frac{215 \times \pi}{3\sqrt{2} \times 296.192}$$

$$\text{or, } \cos \alpha_2 = 0.5375$$

$$\therefore \alpha_2 = 57.486^\circ$$

Now,

$$\text{At } 1800 \text{ rpm, } \alpha_1 = 0^\circ$$

So,

α varies from 0° (at 1800 rpm)
to 57.486° (at 900 rpm)

77) Given,

At rated motor conditions,

Rated armature voltage, $V_{a1} = 500 \text{ V}$

Rated speed, $N_1 = 1500 \text{ rpm}$

Rated current, $I_{a1} = 100 \text{ A}$

Armature resistance, $R_a = 1.1 \Omega$

Now,

The back emf at rated speed is,

$$\begin{aligned} E_{b1} &= V_{a1} - I_{a1} R_a \\ &= 500 - 100 \times 1.1 \\ &= 390 \text{ V} \end{aligned}$$

At operating speed ($N_2 = 1200 \text{ rpm}$),

$$E \propto N$$

$$\therefore E_2 = E_1 \times \frac{N_2}{N_1}$$

$$= 390 \times \frac{1200}{1500}$$

$$\therefore E_2 = 312 \text{ V}$$

Now,

For a 3 phase semi controlled bridge converter,

$$\text{O/p Voltage, } V_o = \frac{3\sqrt{2} V_L}{2\pi} (1 + \cos\alpha)$$

We have,

$$\text{Line Supply Voltage, } V_L = 350 \text{ V}$$

$$\text{Firing angle, } \alpha = 45^\circ$$

Then,

$$V_o = \frac{3\sqrt{2} \times 350}{2\pi} (1 + \cos 45)$$

$$\text{or, } V_o = 403.44 \text{ V}$$

Now, We have,

$$V_o = E_a + I_a R_a$$

$$\text{or, } I_a = \frac{V_o - E_a}{R_a} = \frac{403.44 - 312}{1.1}$$

$$\therefore I_a = 83.13 \text{ A}$$

Now,

In a 3- ϕ semi-converter, each thyristor conducts for 120° ($1/3$ of a cycle)

Then,

1) Average Thyristor Current $[I_T(\text{avg})]$,

$$I_T(\text{avg}) = \frac{I_a}{3} = \frac{83.13}{3} = 27.71 \text{ A}$$

2) RMS Thyristor Current $[I_T(\text{rms})]$,

$$I_T(\text{rms}) = \frac{I_a}{\sqrt{3}} = \frac{83.13}{\sqrt{3}} = \cancel{48.00 \text{ A}} \\ 47.995 \text{ A}$$

3) RMS Source Current,

$$I_s(\text{rms}) = \sqrt{\frac{2}{3}} I_a \\ = \sqrt{\frac{2}{3}} \times 83.13 \\ = 67.88 \text{ A}$$

Now,

W/P Power Factor is given by,

$$pf = \frac{\text{Active Power (P}_{in})}{\text{Apparent Power (S}_{in})}$$

$$= \frac{V_o I_a}{\sqrt{3} V_L I_s (\text{rms})}$$

We have,

$$V_o = 403.44$$

$$I_a = 83.13$$

$$V_L = 350 \text{ V}$$

$$I_s (\text{rms}) = 67.88 \text{ A}$$

Then,

$$pf = \frac{403.44 \times 83.13}{\sqrt{3} \times 350 \times 67.88}$$

$$\therefore pf = 0.815 \text{ (lagging)}$$

2e) Here, given,

220V, DC shunt motor

$$\begin{aligned} \cancel{I_a} &= \cancel{22A} & I_L &= 22A \\ N_1 &= 1000 \text{ rpm} \\ R_f &= 100 \Omega \\ R_a &= 0.1 \Omega \\ N_2 &= 800 \text{ rpm} \end{aligned}$$

In shunt motor, field remains constant. So,

$$E_b \propto N$$

$$\text{or, } \frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1} \quad \text{--- (1)}$$

We have,

$$I_f = \frac{V}{R_f} = \frac{220}{100} = 2.2A$$

Then,

$$I_a = I_L - I_f$$

$$= 22 - 2.2$$

$$\therefore I_a = 19.8A$$

a) Since the load torque is proportional to the speed,

$$\tau \propto N$$

$$\text{or, } \frac{\tau_2}{\tau_1} = \frac{N_2}{N_1}$$

also,

$$\tau \propto \phi \cdot I_a$$

i.e.,

$$\frac{\tau_2}{\tau_1} = \frac{I_{a2}}{I_{a1}}$$

i.e.,

$$\frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1}$$

$$\text{or, } \frac{I_{a2}}{I_{a1}} = \frac{800}{1000}$$

$$\text{or, } I_{a2} = \frac{800}{1000} \times 19.8$$

$$\therefore I_{a2} = 15.84 \text{ A}$$

Let, R_x be the external resistance connected to armature so that, speed reduces to 800 rpm.

then,

$$E_{b2} = V - I_{a2} (R_a + R_x)$$

$$\text{or, } E_{b2} = 220 - 15.84 (0.1 + R_x)$$

$$\therefore E_{b2} = 220 - 15.84 (0.1 + R_x)$$

again,

$$\begin{aligned} E_{b1} &= V - I_{a1} R_a \\ &= 220 - 19.8 \times 0.1 \\ &= 218.02 \text{ V} \end{aligned}$$

then,

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\text{or, } \frac{220 - 15.84(0.1 + R_n)}{218.02} = \frac{800}{1000}$$

$$\text{or, } 220 - 15.84(0.1 + R_n) = 174.416$$

$$\text{or, } 220 - 174.416 = 15.84(0.1 + R_n)$$

$$\text{or, } 45.584 = 15.84(0.1 + R_n)$$

$$\text{or, } 0.1 + R_n = 2.878$$

$$\boxed{\text{or, } R_n = 2.778 \Omega}$$

b) Since the load torque is proportional to the square of speed i.e.

$$T \propto N^2$$

$$\text{or, } \frac{T_2}{T_1} = \frac{N_2^2}{N_1^2}$$

Also,

$$\tau \propto \phi I_a$$

Here ϕ is constant so,

$$\frac{I_{a2}}{I_{a1}} = \left(\frac{800}{1000} \right)^2$$

$$\text{or, } I_{a2} = \left(\frac{800}{1000} \right)^2 \times 19.8$$

$$\therefore I_{a2} = 12.672 \text{ A}$$

Let R_a be the external resistance connected to armature so that speed becomes 800 rpm.

$$E_{b2} = V - I_{a2} (R_a + R_n)$$

$$\therefore E_{b2} = 220 - 12.672 (0.1 + R_n)$$

Similarly,

$$E_{b1} = V - I_{a1} R_a$$

$$= 220 - 19.8 \times 0.1$$

$$= 218.02 \text{ V}$$

Then,

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\text{or, } \frac{220 - 12.672(0.1 + R_n)}{218.02} = \frac{800}{1000}$$

$$\text{or, } 220 - 12.672(0.1 + R_n) = 174.416$$

$$\text{or, } 220 - 174.416 = 12.672(0.1 + R_n)$$

$$\text{or, } \frac{45.584}{12.672} = 0.1 + R_n$$

$$\boxed{\therefore R_n = 3.497 \Omega}$$

$$\text{for i)} \quad R_n = 2.778 \Omega$$

$$\text{ii)} \quad R_n = 3.497 \Omega$$

79) given that,

$$\text{Supply } (V_s) = 400 \text{ V}$$

$$R_a = 0.1 \Omega$$

Motor voltage (torque) constant, $K\phi = 4 \text{ Vs/rad}$

Armature current, $I_a = 150 \text{ A}$ (continuous, ripple free)

Duty cycle, $\delta = 0.6$ (60%)

Chopper losses neglected.

Now,

For a step down (buck) choppers feeding a separately excited DC motor,

$$V_a = \delta V_s$$

$$V_a = E_b + I_a R_a$$

where,

$$E_b = K \phi \omega$$

$$T_d = K \phi I_a$$

Since, the chopper is lossless, input power from the source equals output power delivered to the motor:

$$P_{in} = P_{out} = V_a I_a$$

a) Input power:

Average motor terminal voltage:

$$V_a = \delta V_s = 0.6 \times 400 \\ = 240 \text{ V}$$

Input power,

$$P_{in} = V_a I_a = 240 \times 150 \\ = 36000 \text{ W} = 36 \text{ kW}$$

b) Motor Speed

Back emf,

$$\begin{aligned} E_b &= V_a - I_a R_a \\ &= 240 - 150 \times 0.1 \\ &= 225 \text{ V} \end{aligned}$$

We have,

$$\begin{aligned} E_b &= K \phi \omega \\ \text{or, } \omega &= \frac{225}{4} \end{aligned}$$

$$\therefore \omega = 56.25 \text{ rad/sec}$$

$$\left[\text{In rpm: } N = \frac{60 \omega}{2\pi} = \frac{60 \times 56.25}{2\pi} = 537.148 \text{ rpm} \right]$$

c) Developed Torque,

$$\begin{aligned} T_d &= K \phi I_a \\ &= 4 \times 150 \\ &= 600 \text{ Nm} \end{aligned}$$

Now, For Maximum and Minimum Speed
(ϕ Varying From 10% to 90%)

Since I_a is assumed to remain constant at 150A, only V_a changes with duty cycle.

2) Minimum speed ($\delta = 0.1$)

$$V_a = 0.1 \times 400 \\ = 40 \text{ V}$$

$$E_b = V_a - I_a R_a \\ = 40 - (150 \times 0.1) \\ = 25 \text{ V}$$

Then,

$$\omega_{\min} = \frac{25}{4} \\ = 6.25 \text{ rad/sec}$$

i.e.

$$N_{\min} = \frac{60 \omega}{2\pi} \\ = \frac{60 \times 6.25}{2\pi}$$

$$\therefore N_{\min} = 59.683 \text{ rpm}$$

3) Maximum speed ($\delta = 0.9$)

$$V_a = 0.9 \times 400 \\ = 360 \text{ V}$$

$$E_b = V_a - I_a R_a \\ = 360 - 150 \times 0.1 \\ = 345 \text{ V}$$

Then,

$$\omega_{\max} = \frac{845}{4}$$

$$= 86.25 \text{ rad/sec}$$

i.e.

$$N_{\max} = \frac{60\omega}{2\pi}$$

$$= \frac{60 \times 86.25}{2\pi}$$

$$= 823.627 \text{ rpm}$$

20) Given,

$$V = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$R_f = 250 \Omega$$

$$R_a = 0.2 \Omega$$

$$T_L = 50 \text{ N.m}$$

$$N = 1000 \text{ rpm}$$

$$K_b = 0.8 \text{ V/A - rad/s}$$

$$K_t = 0.8 \text{ N/A}^2$$

$$\alpha_f = 0^\circ$$

Since, both converters are single phase semi-converters, whose average (dc) o/p voltage is,

$$V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

where, $V_m = \sqrt{2} V$

i) Field current,

For the field circuit,

$$\alpha_f = 0^\circ. \text{ So,}$$

$$V_f = \frac{V_m}{\pi} (1 + \cos 0)$$

$$= \frac{2V_m}{\pi}$$

$$= \frac{2\sqrt{2} \times 220}{\pi}$$

$$= 198.06 \text{ V}$$

then,

$$I_f = \frac{V_f}{R_f} = \frac{198.06}{250}$$

$$= 0.792 \text{ A}$$

ii) Firing angle of the armature converter

Since, the motor is separately excited with variable field, torque is,

$$T = K_t I_f I_a$$

$$\begin{aligned} \text{oo, } I_a &= \frac{T}{K_t I_f} = \frac{50}{0.8 \times 0.792} \\ &= 78.914 \text{ A} \end{aligned}$$

Again,

$$\begin{aligned} \omega &= \frac{2\pi N}{60} \\ &= \frac{2\pi \times 1000}{60} \\ &= 104.72 \text{ rad/s} \end{aligned}$$

Then,

$$\begin{aligned} E_b &= K_v I_f \omega \\ &= 0.8 \times 0.792 \times 104.72 \\ &= 66.35 \text{ V} \end{aligned}$$

Then,

Required armature voltage,

$$\begin{aligned} V_a &= E_b + I_a R_a \\ &= 66.35 + 78.914 \times 0.2 \\ &= 82.1328 \text{ V} \end{aligned}$$

then, using relation,

$$V_a = \frac{V_m}{\pi} [1 + \cos \alpha_a]$$

$$\text{or, } 82.13 = \frac{220 \times \sqrt{2}}{\pi} [1 + \cos \alpha_a]$$

$$\text{or, } \cos \alpha_a = -0.17069$$

$$\therefore \alpha_a = 99.83^\circ = 1.742 \text{ rad}$$

Q.3) Power factor of armature converter

For a single-phase semi-converter with continuous, ripple free load current, the rms value of the supply current I_s ,

$$I_s = I_a \sqrt{\frac{\pi - \alpha}{\pi}} = 78.914 \sqrt{\frac{\pi - 1.742}{\pi}}$$

then,

$$= 52.67 \text{ A}$$

We have,

$$pf = \frac{\text{Power } I/P}{V I_s}$$

$$= \frac{V_a I_a}{V I_s}$$

$$= \frac{V_a I_a}{V I_s}$$

$$= \frac{V_a I_a}{V I_s}$$

$$= \frac{82.1328 \times 78.914}{220 \times 52.67}$$

$$220 \times 52.67$$

$$\therefore pf = 0.5594 \text{ (lagging)}$$

32) Here, given,

220V, 50Hz (single phase, full controlled bridge for both armature and field)

$$R_a = 0.2 \Omega$$

$$R_f = 150 \Omega$$

$$K_f = 1.1 \text{ V / (A.rad/s)}$$

$$I_a = 25 \text{ A (continuous, ripple free)}$$

The field current is set to max^m possible value. So, the field converter is operated at,

$$\alpha_f = 0^\circ$$

Also, for armature converter,

$$\alpha_a = 45^\circ$$

54

Now, for a single-phase full controlled bridge converter,

$$V_{dc} = \frac{2V_m}{\pi} \cos \alpha$$

$$V_m = \sqrt{2} V_s$$

$$E_b = K' I_f \omega_m$$

$$T = K' I_f I_a$$

$$V_a = E_b + I_a R_a$$

Now,

We have,

$$\begin{aligned} V_f &= \frac{2V_m}{\pi} \cos \alpha_f \\ &= \frac{2\sqrt{2} \times 220}{\pi} \times \cos 0 \\ &= 198.07 \text{ V} \end{aligned}$$

Then,

$$\begin{aligned} I_f &= \frac{V_f}{R_f} = \frac{198.07}{150} \\ &= 1.32 \text{ A} \end{aligned}$$

Again,

$$V_a = \frac{2V_m \cos \alpha_a}{\pi}$$

$$= \frac{2 \times \sqrt{2} \times 220}{\pi} \times \cos 45^\circ$$

$$= 140.06 \text{ V}$$

Now,

i) Torque developed by the motor,

$$T = K' I_f I_a$$

$$= 1.1 \times 1.32 \times 25$$

$$= 36.3 \text{ Nm}$$

ii) Speed of the motor,

We have,

$$E_b = V_a - I_a R_a$$

$$= 140.06 - 25 \times 0.2$$

$$= 135.06 \text{ V}$$

Then,

Speed (rad/s)

$$\omega_m = \frac{E_b}{K' I_f} = \frac{135.06}{1.1 \times 1.32}$$

$$= 93.02 \text{ rad/s}$$

then,

$$N = \frac{60 \text{ Wm}}{2\pi}$$

$$= \frac{60 \times 93.02}{2\pi}$$

$$= 888.28 \text{ rpm}$$

∴, rms value equals the DC current itself.

Armature side

$$\text{Power}_a = V_a I_a = 140.06 \times 25$$

$$= 3501.5 \text{ W}$$

$$VA_a = V_s I_a = 220 \times 25$$

$$= 5500 \text{ VA}$$

field side

$$\text{Power}_f = V_f I_f = 198.07 \times 1.32$$

$$= 261.45 \text{ W}$$

$$VA_f = V_s I_f = 220 \times 1.32$$

$$= 290.4 \text{ VA}$$

then,

$$\text{pf} = \frac{\text{Power}_a + \text{Power}_f}{VA_a + VA_f} = \frac{3501.5 + 261.45}{5500 + 290.4}$$

$$= 0.6498 \text{ (lagging)}$$

gs Solⁿ,

We have,

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Slip at rated speed (1350 rpm),

$$\begin{aligned} s_1 &= \frac{N_s - N}{N_s} \\ &= \frac{1500 - 1350}{1500} \\ &= 0.1 \end{aligned}$$

For an induction motor, the torque expression is,

$$T = \frac{3}{\omega_s} \times \frac{V_{ph}^2}{\left(R_1 + \frac{R_2'}{s}\right)^2 + \left(X_1 + X_2'\right)^2} \times \left(\frac{R_2'}{s}\right)$$

Then,

$$\begin{aligned} \omega_s &= \frac{2\pi N_s}{60} = \frac{2\pi \times 1500}{60} \\ &= 157.079 \text{ rad/sec} \end{aligned}$$

again,

$$V_{ph,1} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$\begin{aligned} \text{Then, } X_{\text{total}} &= X_1 + X_2' \\ &= 4 + 4 \\ &= 8 \Omega \end{aligned}$$

Then,

$$R_{eq,1} = R_1 + \frac{R_2'}{s_1}$$

$$= \frac{1.5 + \frac{4}{0.1}}{0.1}$$

$$= 41.5 \Omega$$

$$|Z|^2 = R_{eq,1}^2 + X_{\text{total}}^2$$

$$= 41.5^2 + 8^2$$

$$= 1786.25 \Omega^2$$

Then,

Developed torque at 1850 rpm,

$$T_1 = \frac{3}{\omega_s} \times \frac{V_{ph,1}^2 \times (R_2'/s_1)}{|Z|^2}$$

$$= \frac{3}{157.079} \times \frac{230.94^2 \times (4/0.1)}{1786.25}$$

$$= 22.81 \text{ Nm}$$

Now

Since the load torque varies as the square of the speed ($T_L \propto N^2$),

$$\begin{aligned} T_2 &= T_1 \left(\frac{N_2}{N_1} \right)^2 \\ &= 22.81 \times \left(\frac{900}{1350} \right)^2 \\ &= 10.14 \text{ Nm} \end{aligned}$$

At 900 rpm

$$\text{slip } (s_2) = \frac{1500 - 900}{1500}$$

$$= 0.4$$

Similarly,

$$R_{eq,a} = R_s + \frac{R_2'}{s_2}$$

$$= 1.5 + \frac{4}{0.4}$$

$$= 11.5 \Omega$$

$$|Z_2|^2 = R_{eq,2}^2 + X_{tot,2}^2$$

$$= 11.5^2 + 8^2$$

$$= 196.25 \Omega^2$$

then, again,

$$I_2 = \frac{3}{\omega_s} \times \frac{V_{ph,2}^2}{|Z_2|^2} \times \left(\frac{R_2'}{S_2} \right)$$

$$\text{or, } 10.14 = \frac{3}{157.079} \times \frac{V_{ph,2}^2}{196.25} \times \left(\frac{4}{0.4} \right)$$

$$\text{or, } 10.14 = V_{ph,2}^2 \times 0.000973$$

$$\text{or, } V_{ph,2}^2 = 10421.38$$

$$\therefore V_{ph,2} = 102.09 \text{ V/phase}$$

then, Applied line voltage,

$$V_{L2} = \sqrt{3} \times V_{ph,2}$$

$$= \sqrt{3} \times 102.09$$

$$= 176.825 \text{ V}$$

96) Given,

Number of poles, $P = 8$

Supply frequency, $f = 25 \text{ Hz}$

Initial slip, $s_1 = 4\% = 0.04$

Rotor resistance / phase, $R_2 = 0.1 \Omega$

Standstill rotor reactance / phase,

$$X_2 = 0.6 \Omega$$

Additional resistance / phase,

$$R_{add} = 0.5 \Omega$$

Therefore, new rotor resistance is,

$$R_2' = R_2 + R_{add}$$

$$= 0.1 + 0.5$$

$$= 0.6 \Omega$$

Since, the motor is required to deliver the same full-load torque, the torque before and after adding resistance is equal.

We have,

$$\tau \propto \frac{s R_2}{R_2^2 + (s X_2)^2}$$

Hence, initially,

$$\tau_1 \propto \frac{s_1 R_2}{R_2^2 + (s_1 X_2)^2}$$

Let, new slip be s_2

$$\text{and } R_2' = 0.6 \Omega$$

So,

For same torque,

$$\tau_1 = \tau_2$$

$$\text{or, } \frac{s_1 R_2}{R_2^2 + (s_1 X_2)^2} = \frac{s_2 R_2'}{R_2'^2 + (s_2 X_2)^2}$$

$$\text{or, } \frac{0.04 \times 0.1}{0.1^2 + (0.04 \times 0.6)^2} = \frac{s_2 \times 0.6^2}{0.6^2 + (s_2 \times 0.6)^2}$$

$$\text{or, } 0.378 = \frac{0.36 s_2}{0.36 + 0.36 s_2^2}$$

$$\text{or, } 0.13608 + 0.13608 s_2^2 = 0.36 s_2$$

$$\text{or, } 0.13608 s_2^2 - 0.36 s_2 + 0.13608 = 0$$

$$\text{or, } \frac{0.04 \times 0.1}{0.1^2 + (0.04 \times 0.6)^2} = \frac{0.6s_2}{0.6^2 + (0.6s_2)^2}$$

$$\text{or, } \frac{0.004}{0.010576} = \frac{\cancel{0.6}s_2}{0.6^2(1+s_2^2)}$$

$$\text{or, } \frac{s_2}{1+s_2^2} = \frac{0.0024}{0.010576}$$

$$\text{or, } 0.010576s_2 = 0.0024 + 0.0024s_2^2$$

$$\text{or, } 0.0024s_2^2 - 0.010576s_2 + 0.0024 = 0$$

Solving,

$$s_2 = 4.167 \quad \text{or } 0.24$$

(Impossible)
 > 1

i.e.

$$s_2 = 0.24 = 24\%$$

Now,
the synchronous speed is,

$$N_s = \frac{120f}{P}$$

$$= \frac{120 \times 25}{8}$$

$$\therefore N_s = 375 \text{ rpm}$$

Now, the rotor speed is,

$$\begin{aligned} N_r &= N_s (1 - s) \\ &= 375 (1 - 0.24) \\ &= 285 \text{ rpm} \end{aligned}$$

102) Given,

400V, star connected,

3 ϕ , 6 pole, 50 Hz IM

parameters referred to stator,

$$r_1 = r_2' = 1 \Omega$$

$$x_1 = x_2' = 2 \Omega$$

Given,

$$N_{fl} = 950 \text{ rpm}$$

to be braked by plugging

$$\text{Stator to rotor turn ratio} = \frac{2}{3}$$

then,

$$\text{Synchronous speed } (N_s) = \frac{120f}{P}$$

$$= \frac{120 \times 50}{6}$$

$$= 1000 \text{ rpm}$$

$$\omega_s = \frac{2\pi N_s}{60} = \frac{2\pi \times 1000}{60}$$

$$= 104.72 \text{ rad/s}$$

Then,

$$\text{Full load slip } (s_{fl}) = \frac{N_s - N_{fl}}{N_s}$$

$$= \frac{1000 - 950}{1000}$$

$$= 0.05$$

Then,

Full load Impedance,

$$Z_{fl} = \left(r_1 + \frac{r_2'}{s_{fl}} \right) + j(x_1 + x_2')$$

$$= \left(1 + \frac{1}{0.05} \right) + j(2 + 2)$$

$$= 21 + j4$$

$$|Z_{fl}| = \sqrt{21^2 + 4^2}$$

$$= 21.38 \Omega$$

then,

full load current is given by,

$$I_{fl} = \frac{V_{ph}}{|Z_{fl}|} = \frac{400/\sqrt{3}}{21.38}$$

$$= 10.80 \text{ A}$$

then,

$$T_{fl} = \frac{3}{\omega_s} \times I_{fl}^2 \times \frac{r_2'}{s_{fl}}$$

$$= \frac{3}{104.72} \times (10.80)^2 \times \frac{1}{0.05}$$

$$= 66.83 \text{ Nm}$$

3) Initial braking current and torque ratios

During plugging, the phase sequence is reversed while the continues moving forward due to inertia.

So,

→ Initial plugging slip (s_b),

$$s_b = \frac{N_s - (-N_{pl})}{N_s}$$

$$= 2 - s_{pl}$$

$$= 2 - 0.05$$

$$= 1.95$$

→ Initial braking impedance (Z_b),

$$Z_b = \left(r_1 + \frac{r_2'}{s_b} \right) + j(x_1 + x_2')$$

$$= \left(1 + \frac{1}{1.95} \right) + j(2 + 2)$$

$$= 1.513 + j4 \Omega$$

$$|Z_b| = \sqrt{1.513^2 + 4^2}$$

$$= 4.276 \Omega$$

→ Initial braking current (I_b) and ratio,

$$\begin{aligned}
 I_b &= \frac{V_{ph}}{|Z_b|} \\
 &= \frac{400/\sqrt{3}}{4.276} \\
 &= 54.008 \text{ A}
 \end{aligned}$$

then,

Current ratio,

$$\frac{I_b}{I_{fl}} = \frac{|Z_{fl}|}{|Z_b|} = \frac{21.38}{4.276} = 5$$

→ Initial braking torque (T_b) and ratio,

$$T_b = \frac{3}{\omega_s} \cdot I_b^2 \cdot \left(\frac{r_2'}{s_b} \right)$$

$$= \frac{3}{104.72} \times 54.008^2 \times \frac{1}{1.95}$$

$$= 42.85 \text{ Nm}$$

then,

$$\text{torque ratio } \left(\frac{T_b}{T_{fl}} \right) = \left(\frac{I_b}{I_{fl}} \right)^2 \times \left(\frac{s_{fl}}{s_b} \right)$$

$$= s^2 \times \left(\frac{0.05}{1.95} \right)$$

$$= 0.641$$

99) External rotor resistance and new initial braking torque,

to limit initial braking current to $1.5 \times I_{fl}$,

$$|Z_{b2}| = \frac{|Z_{fl}|}{1.5}$$

$$= \frac{21.38}{1.5}$$

$$= 14.253 \Omega$$

We have,

$$|Z_{b2}| = \sqrt{\left(r_1 + \frac{R_{2, total}'}{s_b} \right)^2 + (x_1 + x_2')^2}$$

$$\text{or, } 14.253 = \sqrt{\left(1 + \frac{R_{2, total}'}{1.95} \right)^2 + 4^2}$$

$$\text{or, } 203.148 = \left(1 + \frac{R_{2, total}'}{1.95} \right)^2 + 16$$

$$\text{or, } \left(1 + \frac{R_{2', \text{total}}}{1.95} \right)^2 = 187.148$$

$$\text{or, } 1 + \frac{R_{2', \text{total}}}{1.95} = 13.68$$

$$\text{or, } \frac{R_{2', \text{total}}}{1.95} = 12.68$$

$$\text{or, } R_{2', \text{total}} = 24.726 \, \Omega / \text{phase}$$

Now,

stator-referred external resistance,

$$R_{\text{ext}}' = R_{2', \text{total}} - r_2'$$

$$= 24.726 - 1$$

$$= 23.726 \, \Omega / \text{phase}$$

Now, actual external resistance on rotor side is,

$$R_{\text{ext}} = R_{\text{ext}}' \times \left(\frac{N_2}{N_1} \right)^2$$

$$= 23.726 \times \left(\frac{3}{2} \right)^2$$

$$= 53.383 \, \Omega / \text{phase}$$

New initial braking torque (T_{b2}),

$$T_{b2} = \frac{3}{w_s} \cdot (1.5 I_{fl})^2 \times \left(\frac{R'_{2, total}}{s_b} \right)$$

$$= \frac{3}{104.72} \times (1.5 \times 10.80)^2 \times \left(\frac{24.726}{1.95} \right)$$

$$= 95.33 \text{ Nm}$$

Then,

~~Ratio to full load torque~~

~~$$\frac{T_{plug, new}}{T_{fl}} = \frac{95.33}{42.85} =$$~~

Ratio to full load torque,

$$\frac{T_{b2}}{T_{fl}} = \frac{95.33}{66.89}$$

$$= 1.426$$

$$\therefore T_{b2} = 1.426 T_{fl}$$